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Envisioning Equitable Mathematics Education: Promoting relevance and engagement

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Foreword

Mathematics serves as a gateway to majority careers, especially in Natural Sciences, Engineering, Technology and Accountancy. Access and acquittance in these fields rely on competence in both basic and advanced mathematical skills and knowledge. Equitable mathematics education ensures that all students, regardless of race, gender, socioeconomic status, language, or ability, have access to meaningful and empowering math learning experiences. The goal is to make math relevant, engaging, and liberatory and not just procedural or performative. In order to promote equitable mathematics education, it is essential to prioritise relevance and engagement in the teaching and learning process. This means providing opportunities for students to see the practical applications of mathematics in their everyday lives and to engage with the material in meaningful ways that resonate with their interests and experiences.

One way to promote relevance is to incorporate real-world examples and contexts into the mathematics curriculum. This could involve using data and statistics from current events, discussing how mathematics is used in various professions, or exploring historical contributions of underrepresented groups to the field of mathematics. By connecting the material to students' lives and interests, educators can help them see the value and importance of mathematical concepts.

Engagement can also be fostered through active and collaborative learning experiences. This could involve group projects, hands-on activities, and interactive technology tools that allow students to explore and apply mathematical concepts in a more dynamic and engaging way. By creating a classroom environment that encourages curiosity, exploration, and creativity, educators can help students develop a deeper understanding and appreciation for mathematics.

Overall, by promoting relevance and engagement in mathematics education, educators can create a more inclusive and equitable learning environment where all students have the opportunity to succeed and thrive. With a focus on connecting mathematics to real-world contexts and fostering active engagement in the learning process, we can help students develop the critical thinking skills and problem-solving abilities needed to navigate our increasingly complex and data-driven world. The current theme, 'Envisioning Equitable Mathematics Education: Promoting Relevance and Engagement" was conceptualised against this background.

One important way to foster relevance and engagement in mathematics education is through real-world applications. Studies have indicated that students are more inclined to connect with and comprehend the relevance of mathematical topics when they can see how they apply in real-world settings. For instance, instructors may demonstrate to their students how mathematics is applied in real-world situations by using examples from areas like finance, engineering, and medicine.

Teachers can encourage greater engagement with the subject by giving students more freedom and opportunities to investigate mathematical concepts independently. For instance, instructors may use project-based learning exercises that enable students to use mathematical principles to solve real-world issues and problems.

Promoting relevance and engagement in South African mathematics classrooms is crucial for enhancing learner performance, decreasing dropout rates, and making math more meaningful by connecting curriculum content to students' real-life experiences and future aspirations, thereby fostering intrinsic motivation and active participation. A number of presenters are engaging with some of these issues that arise from those and strategies that can be used to address them. Our panellists and plenary speakers were also invited to address issues affecting quality mathematics teaching and learning from various perspectives that include higher mathematics learning, teacher education, international perspective, curricular issues and learners support material. The workshops that form part of these proceedings offer hands- on experiences through which facilitators share successful strategies and activities that they have successfully developed to improve the quality of mathematics teaching and learning in different phases and context.

This congress, like any other, will hopefully inspire you to take up the challenges of quality mathematics teaching and learning beyond the five days in which the activities unfold. You will continuously use the two volumes as a critical resource material in your classrooms, seminars, workshops, planning sessions, and so on.

Lastly, we remain thankful for the presenters and their reviewers for their immense contribution to the 30th Annual National Congress of the Association for Mathematics Education of South Africa. We are forever inspired! Enjoy!

Review Process

The papers accepted for publication for the 2025 AMESA Congress were subjected to blind peer review by at least two experienced mathematics education reviewers. The academic committee considered the reviews and made a final decision on the acceptance or rejection of each submission, as well as changing the status of submission. Authors of accepted submission were given the option of not to have their accepted long papers published in the AMESA 2025 Proceedings, to keep open the possibility to submit it to a journal. They were requested to submit an extended abstract rather than their full submission, and this extended abstract will be published in the Proceedings for publication.

Number of submissions: 102
Number of plenary paper submissions: 6
Number of long paper submissions: 34
Number of short paper submissions: 5
Number of workshop submissions: 24
Number of 'How I teach' paper submissions: 14
Number of poster submissions: 1
Number of Maths Market: 20
Number of submissions accepted: 96
Number of submissions rejected: 4
Number of submissions withdrawn by authors: 2

We thank the reviewers for giving their time and expertise to reviewing the submissions.

Reviewers:

Benadette Aineamani	Chipoi Mangwiro	Benjamin Tatira
Alex Jogymol	Mildret Ncube	Msebenzi Rabaza
Mogege Mosimege	Wisani Hlungwani	Tšhegofatšo Makgakga
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ENGAGING LEARNERS IN SOUTH AFRICAN MATHEMATICS CLASSROOMS THROUGH CULTURALLY RELEVANT PROBLEM-SOLVING

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Maths Centre Incorporating Sciences

Abstract

South Africa has a maths crisis in the education sector; children are failing maths at alarming rates, and few learners are engaging with maths at high levels. Lindsay Vally, a local expert with a hard-won perspective on mathematics education in South Africa from attending a dynamic primary learning school, argues instead that engagement in a subject such as mathematics can unify learners who come from diverse cultural backgrounds and create the kind of positive experience meaningful in any subject. Here, we explore culturally relevant problem-solving as a way of addressing the disconnection between mathematical concepts and the lived experience of the learners. When teachers contextualize maths by using real situations, indigenous expertise, and context-based examples in solving problems, the students' attitudes, understanding, and application of maths improve. The paper explores the theory behind culturally relevant teaching (CRT) in mathematics, namely, how CRT is enacted concretely in South African classrooms, and presents empirical data that indicates the approach can engage learners in scholarship and achievement.

Introduction

Mathematics has eased quite a bit a solo decontextualized subject, which resulted in disengagement from learners. Socioeconomic disparities and linguistic diversity contribute to varying levels of comprehension in South Africa. Yet, culturally relevant pedagogy (Ladson-Billings, 1995) has long argued that embedding mathematics within the cultural and social contexts of learners provides opportunities for developing motivation as well as understanding. Common Approaches to Teaching May not Connect with Learners' Cultural Backgrounds This produces a gap between the world they know and its interaction with the world of math. One way of addressing this disconnect is through culturally relevant problem-solving, which attempts to align mathematics more closely with students' lives and experiences. We examine the impact of culturally relevant problem-solving on engagement in South African mathematics classrooms. It examines the approaches wherein the culture of the learner, the use of mathematics in the real world, and indigenous systems of knowledge in the community are all brought to bear on attainment in mathematics.

A Conceptual Framework: Culturally Responsive Teaching in Mathematics

This study is grounded in Ladson-Billings' (1995) theory of Culturally Relevant Teaching (CRT). CRT calls for pedagogy that recognizes, affirms, and incorporates students' culture into the classroom. In mathematics teaching, this means:

- Apply local real-life examples for students' communities.
- Integrating indigenous mathematical concepts and historical contributions
- Promoting collaborative problem-solving, which can resonate with communal learning traditions This orientation reflects Vygotsky's (1978) sociocultural theory, which understands learning as social and informed by culture. Mathematics is more meaningful when they can draw and make sense of it in those experiences.

Learning Mathematics in South Africa: Challenges

Weak performance in national and international assessments (TIMSS, 2019; DBE, 2021). Less contextualization of maths questions. Barriers to comprehension are related to language use (Setati & Adler, 2000). Divergence in socioeconomic status impacting access to adequate quality of education (Spaull, 2013)

Activities on Culturally Relevant Problem-Solving in South African Classrooms

To take culturally relevant problem-solving to the next level, educators need to contextualize and ground the content and the learners ' daily lives. In South African classrooms, the following strategies have proven beneficial:
A few examples of Graphing Stories and Contextualizing Mathematical Problems. Mathematics problems should portray situations encountered in real life, like:

- Realistic financial literacy scenarios rooted in local markets and trading practices
- Measurement and geometry questions about traditional home construction and beadwork patterns
- Statistics and probability principles examined by local sports (i.e., soccer match data)

ii. Integrating Indigenous Knowledge Systems Mathematics has a deep tradition in South African indigenous cultures. For example:

- Symmetrical patterns in Ndebele house paintings and Zulu beadwork demonstrate principles of geometry.
- Alternative numeral systems exist in African tallying and counting.
- Concepts of measurement and ratios can be taught using agricultural and architectural practices

iii. Language and Multilingual Perspectives It is not the case for many learners in South Africa where mathematics is not taught in their home languages, resulting in struggling with conceptual understanding. Some strategies to mitigate this include:

- Translating mathematics key terms in local context: multilingualism
- Promoting switching to help understanding

- Offering problem-solving bilingual resources

iv. Cooperative and Inquiry-Based Learning In South Africa, learners are used to seamlessly learning together. Educators can foster group discussions and problem-solving exercises that emulate traditional knowledge-sharing methods. Working on solving real-world problems through collaboration and critical thinking fosters engagement and deeper learning.

What Does the Research Say?

Research indicates that embedding cultural relevance within mathematics supports improved learner engagement and conceptual understanding (Nasir et al., 2008; Mosimege, 2017). Culturally relevant problem-solving approaches are illustrated by case studies conducted in both rural and urban schools in South Africa. A study in rural Limpopo in 2021 found that students who were taught using contextualized problem-solving techniques scored 15% better than those taught using conventional methods in mathematics. In another case study at a township school in Gauteng, participation increased and learners become more excited about mathematics when learners solved problems that reflected their real-life experience, such as determining household budgets or analyzing rainfall data.

Challenges and Recommendations

While this approach does have advantages, there are some drawbacks to implementing culturally relevant problem solving, such as:

- No training of teachers in CRT methodologies
- Lack of sufficient culturally responsive mathematics books and materials
- Curriculum limitations prioritizing standardized testing over contextualized learning.

To overcome this problem, the following suggestions have been made:

1. Professional development: Training programs should start teachers off on the right foot with culturally relevant teaching strategies.
2. Curriculum Adaptation: Education policymakers must permit local mathematical content.
3. Resource Development: Engaging mathematicians in writing culturally responsive mathematics textbooks.
4. Cultural Context: While not all students in a classroom share the same culture, it is wise for schools to include local communities in developing problem-solving scenarios that reflect the academic challenges in a particular local context.

Conclusion

Problem-solving in culturally relevant ways presents a potential solution to the gap between mathematics education and the realities of learners' lived experiences in South Africa. Incorporating cultural contexts, indigenous knowledge, and real-world applications into mathematics can enhance the engagement and relevance of the subject matter for students. With the right teacher training, curriculum changes, and resource creation, this approach can be effectively integrated, even though challenges will still exist. Future studies should examine longitudinal effects on student performance and strategies for scaling culturally relevant mathematics teaching across a wide variety of educational environments.

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MATHEMATICS TEACHERS' PERCEPTIONS OF USING AI IN THE CLASSROOM: THE CASE OF GAUTENG PROVINCE IN SOUTH AFRICA

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Introduction and research focus

The insurgence and incorporation of powerful and revolutionary technologies like artificial intelligence (AI) in mathematics education demands a conscientious and meticulous re-aligning of the mathematics curriculum, its aims and objectives, delivery approaches, and assessment policies across the whole education spectrum. Despite the central and the critical role that mathematical knowledge plays in driving modern-day innovation and advancements, research evidence shows that globally, many learners still struggle in learning this subject (Mohamed et al. 2022; Opesemowo & Adewuyi, 2024). Surprisingly, the learning difficulties are still evident in mathematics besides significant strides made in terms of research and mobilisation of resources for technology integration in teaching and learning. Studies have attributed the learning challenges in mathematics to various reasons, key among them, insufficient foundational knowledge, ineffective teaching techniques that do not cater to diverse learning styles, and a lack of personalised learning support (Egara & Mosimege, 2024). Evidence from scientific literature suggests that a close connection exists between AI and mathematics education (ME) (Van Vaerenbergh, & Pérez-Suay, 2022). Scholars argue that both AI and ME are concerned with developing reasoning capacities driven by the rules of logic. While this parallel relationship exists, humans and machines clearly carry out these tasks in completely different ways. Harnessing the affordances and potentialities of AI into the mathematics classroom requires not only orienting teachers on how to use it, but to also accept and have a positive perspective towards the technology.

While the teachers' attitudes and expectations lie at the core of successful implementation of such novel innovations, research has shown that teachers' responses to the introduction of any technological system will always vary depending on their pedagogical philosophies, teaching experiences, and technological efficacy, which subsequently influence their desire to accept or resist the new technology (Ayanwale et al., 2024; Funda & Mbangeleli, 2024). As a result, this study is undertaken on the premise that before escalating an AI-driven pedagogical support system, the teachers' understanding, perceptions and expectations must be known, so that professional development gaps may be identified and filled, before investments towards AI eventually become a wastage. The study seeks to answer the following research questions:

1. What is the mathematics teachers' level of understanding of AI systems and applications for teaching and learning?
2. What are the mathematics teachers' perspectives and expectations on the use of AI systems and applications in mathematics education?
3. Are the mathematics teachers' perspectives and usage of AI influenced by variables such as age, experience, gender and qualification?

Motivation of study

Efforts are increasingly being made globally to infuse artificial intelligence (AI) in teaching and learning. Like any other new pedagogical technology, the level of acceptance and successful adoption depends on the readiness and willingness of the practitioners who use the tool. Whether we like it or not, the use of AI has immersed us in a social ecosystem surrounded by sophisticated, fascinating and dynamic technologies (Mohammed et al. 2022). Teachers are confronted with students whose exposure to technology has greatly influenced their learning styles and patterns. Technology has naturally produced a digitally oriented generation capable of learning, discovering and exploring new knowledge without waiting for the so-called more knowledgeable members of society to educate them. Failure by education systems, for whatever reasons, to incorporate these emerging technologies in teaching and learning contexts renders the school system detached and incompatible with real society. For example, researchers have noted that most methods and strategies generally used in traditional teaching still rely on direct instructional techniques such as recitation, dialogue, and discussion, which have which have fallen out of favour with learners and fail to arouse their interest and motivation towards learning (Fannakhosrow et al. 2022; Tashtoush et al. 2023).
AI is more novel and revolutionary in the sense that it reconfigures the way technology is used in education by providing exciting tools, systems, and methods that arouses and capture the interest of this new generation of learners (Abedi, 2023; Mhlongo et al. 2023). When integrated effectively, AI-flavoured learning is believed to enhance comprehension, discovery, and problem-solving, which are important pillars of the goals of mathematics education. AI Technology makes available the resources, facilities, and computing power that would otherwise have been inaccessible or unavailable (Ahmad et al. 2022; Fannakhosrow et al. 2022). Unfortunately, particularly in African education systems, many teachers and school administrators may not have the experience with AI-based learning support and may merely view it as one of the advanced technologies for the private industry. Research in the South African context has shown that AI technologies have the potential to improve personalized learning experiences, promote learner engagement and boost academic performance, especially after traditional teaching methods have become incompatible with the emergence of intelligent systems and applications (Funda & Mbangeleli, 2024; Sikhakhane, Govender & Maphalala, 2021). Thus, this study becomes very handy as it paves the pathway for using AI technology to address the academic challenges and shortcomings that are persistently evident in mathematics education in South Africa.

The technology acceptance model (TAM)

According to the Technology Acceptance Model (Davis,1985), the use of any technological system is a response that can be explained or predicted by two main factors. The first factor is the *'perceived usefulness'* of a technological tool (Davis, 1985). Perceived usefulness is the degree to which a potential user believes that using a particular system would enhance his or her job performance. In a school setup, for example, if teachers are not convinced or do not believe that using a digital innovation will make their teaching more effective and improve learning outcomes, then they are unlikely to use it. The second factor identified in the model is *'perceived ease-of-use*'. Perceived ease of use is explained as the degree to which a person believes that using a particular system would require less effort (Davis, 1989). This depends on the extent to which a targeted user has the technical competency to use or operate a new technology.

Research has shown that lack of technical competency as a barrier to technology adoption and integration in education (Graham, Stols, & Kapp, 2020) can be addressed by increasing availability and access to technological resources and teacher development. However, what research also tells us is that perceived usefulness is a more difficult factor to mitigate. Teachers' own beliefs and perspectives about the relevance and usefulness of any technological device to learning are some of the strongest barriers and pose the greatest threat to the use of technology for teaching and learning (Ertmer, 1999, 2012). The TAM provided the theoretical lens for this study in explaining the teachers' understanding, and perceptions of AI in relation to its perceived role as a novel pedagogical tool that contributes to improved learning outcomes.

Literature review

Conceptualising ai as a digital educational innovation

The term AI was first used by McCarthy (2006) to mean making a machine behave in ways that would be termed 'intelligent' if a person were so acting. Many scholars have since came up with numerous definitions of AI. Akgun and Greenhow (2021) believe that AI is not limited to specific forms of technology, rather it refers to technology, software, methodologies, and computer algorithms utilized to solve human-related problems (Limna et al. 2022; Aldarayseh, 2023, Chen et al., 2020). Unlike traditional computer technologies, which provide a fixed sequence without responding to the user's needs and knowledge, AI attempts to collate patterns of collected information (e.g., student understanding and errors) and make reasonable deductions to facilitate decision making. A systematic review by Mohammed et al. (2022) revealed that AI brings numerous benefits to the educational sector. One of the most notable is personalised education. They argue that AI can provide individualised learning plans for each student, thereby addressing their needs and learning styles. Additionally, Stressing on the same claim, Funda & Mbangeleli (2024) established that AI can enhance learning experiences by providing interactive and engaging content. The novelty of AI as a technology lies in its capacity to build systems to think and act like humans with the ability to achieve goals. Among its most notable capabilities is the innovation's ability to teach and make decisions (Wardat et al. 2024). By simulating human intelligence, AI technologies are designed to think, learn, and make decisions. Inevitably, such AI affordances present pedagogical challenges for educators. For example, if machines can think, reason and make decisions on behalf of the learners, how then can these machines be used to develop thinking and reasoning capacities in humans?

AI has triggered various responses and perceptions in society. Studies have reported perceived challenges, problems and obstacles encountered by educators in using AI systems and applications, which subsequently obstruct their willingness to use them (Abedi, 2023; Al-Mahdi & Majdi, 2021; Hwang, 2022). Some critics consider AI in education as wrong and misplaced because these machines were believed to take over human tasks (Neri & Cozman, 2020). Such risk perception of AI negatively influence consequences related to the variety of applications of AI as a technology. Other scholars argue that the risk perception emerged from defining AI as machine intelligence (Poole et al., 1998), without qualifying the artificiality's boundaries, and failure to differentiate AI from human intelligence (Cope et al., 2020). Consequently, implementing AI in contexts like South African schools is likely to present important social and ethical complications. These include issues of data privacy, addressing bias, and ensuring access and equality in academic settings.

Teacher Perceptions of using AI in education

Research suggests that AI adoption in the classroom has not been fully implemented because many teachers still have a negative attitude toward technology and choose not to use it (Shin & Shin, 2020; Hoorn et al. 2021; Mohammed et al. 2022). The key reasons put forward for this are teacher anxiety in using new technologies (Wang, 2020) and their reluctance to step out of their pedagogical comfort zone (Yeonju et al. 2022]. Some studies concluded that the negative perception teachers hold against AI emanated from the misconceptions which arose as a result of AI disseminated mainly through the media and science fiction (Magdy, 2021). This led some teachers to regard AI as an occupational threat that would replace their jobs rather than as a support that can enhance learning and instruction (Obeidat, 2022). Depending on their unique educational contexts, teachers have varying expectations on what AI should do for them. In some cases, teachers expect AI to provide a more effective teaching and learning process through digitalized learning material and multimodal human-computer interactions (Obeidat, 2022), while others expect it to resolve various learning difficulties of students, catering to their needs despite large class sizes (Hwang, 2022).

Some teachers have been critical of AI use in schools pointing to some of the unintended consequences of heavy dependency on these powerful tools (Jarvis et al. 2022). Jankvist et al. (2019) summed up the perceived negative effects as loss of distinctive features of concept formation, a consequential reclassification of mathematical objects, instability of AI solutions as objects, and prevailing a posteriori reasoning on students behalf when relying solely on AI in their mathematical work. Despite these critical perspectives, the introduction of AI is widely believed to offer greater educational affordances beyond efficiency. What is clear is that education systems must evade the danger of accelerating the introduction and exposure to certain AI systems before teachers and learners were ready for it. This study is undertaken on the premise that before an AI support system can be successfully implemented in any education system, teachers' understanding, perceptions and expectations must be known so that professional development gaps may be identified and filled, before investments towards AI eventually become a wastage.

Methodological aspects

The study adopted the descriptive analytical approach. This methodology suits the study's aim of gathering data to comprehensively describe mathematics teachers' level of understanding of AI, their perspectives, experiences and challenges in using AI in their teaching.

Study sample and Instruments

The study targeted high school mathematics teachers in the Gauteng province of South Africa. Gauteng province was conveniently chosen because it is one of the leading provinces in the country in terms of ICT integration in schools. High school teachers were most appropriate for the study because they deal with more mature learners who are afforded better access to digital gargets and other technologies. A questionnaire was developed to find out mathematics teachers' understanding, perspectives and experiences on using AI systems and applications in their teaching. The questionnaire items were developed based on previous studies and theoretical literature on related variables. The questionnaire items were shared among two dimensions. The first dimension focussed on the teachers' bio-graphical data, which included their age, qualification and experience. The second set of questions focussed on the teachers' understanding of AI systems and applications in teaching, their perspectives, and experiences in using AI for teaching. All items were designed using a Likert scale with five response options: Strongly Agree, Agree, Neutral, Disagree, and Strongly Disagree, where each option was given a numerical rating from one to five, respectively. A team of university lecturers with expertise in the field of educational technology, curricula, and teaching methods were engaged to provide feedback on the research tool. Their input were incorporated to fine tune the instrument until the final version of the tool was achieved. A pilot study was undertaken with a group of 30 teachers to establish the reliability of the instrument. The Cronbach Alpha Coefficient was calculated for each item on the scale, as well as for the first and second domains and the scale as a whole. The Cronbach Alpha coefficients ranged between 0.77 and 0.89 for each item.

Ethical considerations

The study is being undertaken with strict adherence to the principles of confidentiality and anonymity to ensure ethical research. The identities of the participating teachers and all related information are handled with utmost discretion. The participants completed an online survey, and extreme caution will be taken not to attach any uniquely identifying information to data sets, to ensure that no one, not even the researchers, could link data back to any participant. To ensure that confidentiality is uncompromised, the privacy of participants is protected through careful techniques during data collection and analysis, and when reporting, to ensure that participants cannot possibly be identified as being associated with the research (Mertens, 2010). The participants were informed that their participation in the study was completely voluntary, and they could opt out of the study at any time. No participant names are used during data analysis procedures. The team that reviewed the survey instrument ensured that the wording of the question items remained respectful and not personalised, further enhancing the objective nature of this research. Potential threats to the study's integrity included possibly dishonest feedback from participants, and researcher bias. To minimise bias, measures were introduced to prevent the researcher's personal views, reflections and conclusions to become research elements. Written permission to undertake the study in Gauteng schools was granted by the Gauteng Department of Education (GDE). Having satisfied all the ethical considerations the ethics committee of the university that the authors are affiliated with issued the ethical clearance certificate for the study.

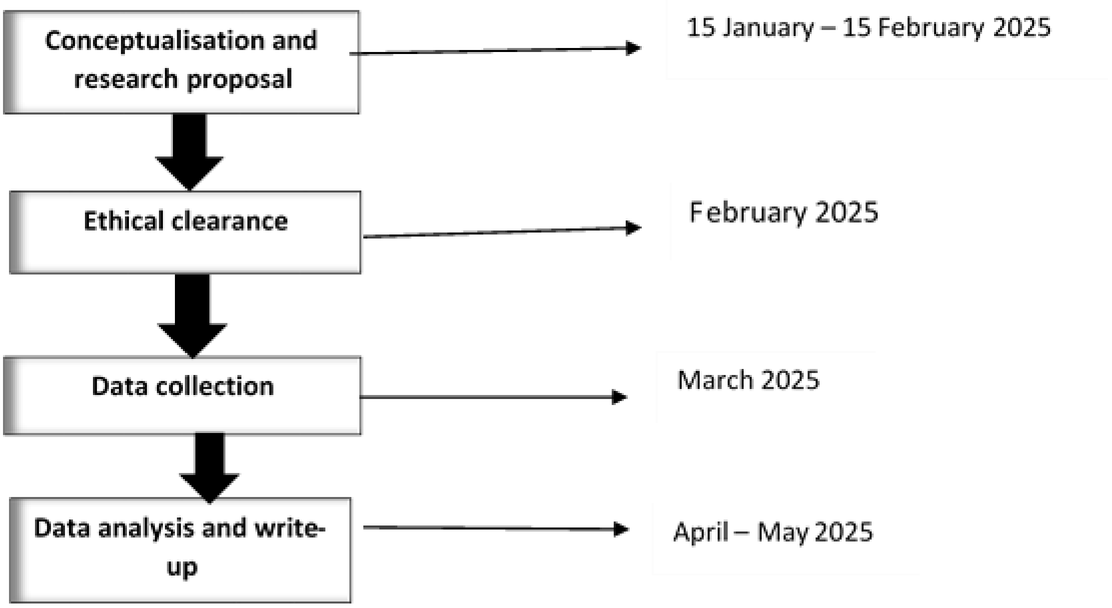
Data analysis plan

Survey data will be analysed using SPSS v25. Incomplete surveys with one or two missing responses will be considered by using an average of other responses in the subscale, while those with more than two missing responses to items in each subscale will be removed from the analysis. The average score across each subscale will be calculated to give an index of adjustment within each domain. Cronbach alpha for the questionnaire constructs will be determined. Pearson's correlation coefficients will be used to establish the underlying correlations between the sub-domain questions. As an example of structural equation modelling (SEM), the confirmatory factor analysis (CFA), which seeks to find out patterns in latent constructs will be used. Using exploratory factor analysis, the factors that influence teachers' perspectives on usage of AI will be unveiled. The t-test and one-way analysis of variance test (One-Way ANOVA) will be used to detect differences between the means in the responses of the study individuals.

Timeline

The study is being undertaken following the time line shown below.

FIGURE 1: Timeline for the study



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EXPLORATION OF THE EXPERIENCES OF MATHEMATICALLY GIFTED HIGH SCHOOL LEARNERS IN A FORMAL EDUCATION SETTING

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Abstract

This interpretive case study explores how formal schooling environments; teacher strategies and peer interactions shape the mathematical development of eight gifted learners (ages 15–18) at a private Western Cape high school. Participants were selected through purposive sampling based on their outstanding results in the mathematics Olympiad competition. Structured interviews were analysed thematically to identify patterns in learners' perceptions regarding opportunities, challenges, and support. The findings suggest that a genuine passion for mathematics and the relevance of real-world problems play a crucial role in sustaining engagement. Conversely, feelings of boredom due to insufficient challenge or frustration stemming from excessive difficulty can hinder progress. Critically, teachers who provide advanced problems and clear scaffolding build confidence, and peer collaboration through shared problem-solving fosters persistence and mathematical identity. These insights underscore the importance of differentiated instruction, enrichment pathways and structured peer networks to meet the unique cognitive and psychosocial needs of mathematically gifted learners. By linking theory and practice, the study offers concrete recommendations for educators seeking to optimise learning experiences and fully realise gifted learners' potential.

Introduction

Do mathematically gifted high school learners experience boredom in traditional classrooms? How does formal education influence their development? In well-resourced schools, such learners are often fast-tracked to maintain academic challenge. However, despite accelerated learning, questions remain as to whether these learners are acquiring deeper problem-solving skills or more effective mathematical methods. This study seeks to explore the experiences of mathematically gifted learners within a formal educational setting. Formal education is defined as a hierarchically graded curriculum, age-based levels, prescribed learning outcomes, and accreditation by national education authorities. This research draws a distinction between giftedness and talent. Giftedness refers to inherent, significantly above-average potential in one or more domains, whereas talent is demonstrated performance shaped by psychosocial and environmental influences (Gagné, 2020). Subotnik, Olszewski-Kubilius, and Worrell (2011) conceptualise giftedness as the upper range of talent, marked by high proficiency. They explain that while giftedness is potential in childhood, it becomes defined by achievement in later life, and is shaped by cognitive, social, and emotional factors. Mathematical giftedness refers to an exceptional aptitude for mathematics well beyond typical age or grade-level expectations

(Singer, Sheffield, Freiman, & Brandl, 2016). Such individuals often display intuitive understanding, rapid problem-solving abilities, and deep interest in mathematical exploration. Sternberg (2018) emphasises that mathematical giftedness is not merely being good at maths, but involves a distinct, exceptional capacity that can lead to advanced achievement in mathematics-intensive fields. Sriraman (2003) argues that problem-solving tasks are key to developing higher-order mathematical thinking and suggests a strong link between giftedness and problem-solving ability. Mathematically gifted learners tend to interpret problems from multiple perspectives and respond with innovative solutions. Parish (2014) supports this view, noting that these learners diverge significantly from peers in their mathematical reasoning, understanding, and learning approaches. In light of this, this study investigates how mathematically gifted high school learners experience and navigate formal education. By exploring their perspectives, the research aims to understand whether traditional educational structures meet their intellectual needs and foster their potential.

Significance of the study

This study makes numerous significant theoretical and practical advances to the teaching of gifted learners. It highlights how a combination of intrinsic desire, appropriately challenging assignments, and social support encourages engagement by lifting the voices of high-achieving learners in a South African culture with ample resources. These observations expand on Gagné's (2004) model by showing the exact ways in which peer cooperation and teacher scaffolding combine to convert potential into performance. This study also has significant implications for curriculum development in mixed-ability settings. An equalising approach in many South African schools may unintentionally marginalise gifted learners, resulting in their isolation or lack of stimulation. The results imply that learning opportunities can be more effectively matched with the interests and readiness of gifted learners through differentiated education, which includes open ended problem-solving, compressed curriculum, and tiered assignments. Incorporating organised peer-learning networks like Olympiad clubs or problem-solving circles offer a reasonable way to promote mathematical confidence and teamwork. This study seeks to better understand how educational settings shape the experiences, motivation, and development of mathematically gifted learners.

Theoretical framework

Albert Bandura's Social Cognitive Theory, which highlights the interaction between behavioural, environmental, and personal factors, known as triadic reciprocal causation, it is the foundation of this work (Bandura, 1986). Individuals watch, understand, and react to their social surroundings, influencing their growth through motivation, self-control, and perceived self-efficacy. According to social cognitive theory, peer relationships, learning environments, and social circumstances all have a significant impact on learners' academic performance (Burney, 2008). By separating giftedness (natural ability) from talent (acquired abilities), Gagné's Differentiated Model of Giftedness and Talent (2004) enhances this paradigm and emphasises the importance of environmental and psychosocial triggers in turning potential into achievement. Gagné recognises giftedness in a variety of fields and emphasises the value of developing these skills. Together, the framework, Social cognitive Theory, support the view that mathematically gifted learners benefit from cognitively stimulating and socially supportive environments. However, in mixed-ability classrooms, gifted learners may face social isolation and a lack of challenge, limiting their development (Neihart, 2002). Social cognitive theory's emphasis on modelling, feedback, and self-efficacy provides a useful lens for examining these dynamics. By applying these theories, this study seeks to better understand how educational settings shape the experiences, motivation, and development of mathematically gifted learners.

Methodology

The study was conducted in a private high school in Cape Town, South Africa's affluent Southern Suburbs. The school was chosen for its accessibility and for implementing the United States Common Core State Standards and place an emphasis on knowledge application, critical thinking, and problem solving. In accordance with Subotnik et al. (2011)'s criterion of giftedness, eight mathematically gifted learners, ages 15 to 18, were purposefully selected based on their enrolment in accelerated mathematics courses and eligibility for advanced rounds of national Mathematics Olympiads. Academically diverse placements were made possible by this ability-based course structure, a grade 9 learner in the same class as a grade 12 learner.

To reduce academic bias, each participant participated in a 20 to 30 minute semi-structured interview after school hours after end-of-term examinations. Interviews were audio recorded, verbatim transcribed, and anonymised with informed consent. Rich qualitative data was produced by using open-ended questions, which allow for a more thorough examination of participants' support networks, classroom difficulties, and self-directed learning (Creswell & Poth, 2018). To interpret and synthesise these transcripts into logical accounts of gifted learners' experiences in this formal educational setting, data analysis required a methodical, multifaceted procedure that went beyond simple organisation (Cohen, Manion & Morrison, 2007; Fraenkel & Wallen, 2005).

Findings

Challenge and Self-confidence

These talented learners have a strong positive relationship with mathematics, and they frequently talk about experiencing "flow" and being captivated by difficult subjects like calculus. They describe the topic as engaging and significant when it relates to the real world. Their enthusiasm may sometimes turn into frustration; too easy tasks make them agitated and uninspired, while content beyond their current comprehension can cause real difficulty. Learners frequently give their teachers credit for establishing the intellectual foundation that fosters competence and confidence by posing challenging tasks, providing clear explanations, and suggesting enrichment activities. When learners are given exactly the right amount of stretch, enough challenge to encourage growth without discouraging them, that confidence rises even more.

Peer Collaboration and Encouragement

Peers offer a dynamic environment for collaboration and support, helping to improve problem-solving techniques by contrasting various methods, pointing out one other's mistakes, and encouraging one another to keep going. In addition to encouraging perseverance, friendly competition promotes a sense of community and a common mathematical identity. Classmates keep each other motivated and involved by bringing abstract ideas to life using debate and group problem-solving.

Support and Learning Driven by Passion

Continuous involvement is maintained in a nurturing learning environment that encourages resources for independent study, mentorship, and feedback. Whether through online courses or extracurricular reading, encouraging kids to research topics independently boosts their self-esteem and cultivates a lifelong curiosity. Teachers set the pace and offer the framework for more complex inquiry, while peers stimulate these independent investigations. In the end, these learners' experiences of formal education are shaped by the interaction of inherent desire, well calibrated difficulties, and a supportive community, which allows them to succeed academically and to grow into resilient, self-assured mathematicians.

Discussion

The findings of this study confirm that in formal educational contexts, mathematically gifted learners encounter an intricate structure of difficulty, support, and enjoyment. When working on more complex subjects, they frequently express "flow" and interest, which is consistent with Singer et al. (2016)'s description of mathematically gifted learners' intuitive comprehension and quick problem-solving. The dual sense of boredom under low challenge and frustration under excessive difficulty, however, supports Sriraman's (2003) contention that problem-solving exercises need to be precisely regulated to promote higher order thinking without causing disengagement. According to Bandura's Social Cognitive Theory (1986), peer modelling provides vicarious learning that maintains motivation and fosters a sense of community, while positive reinforcement from peers and teachers acts as moments of mastery.

Teacher scaffolding emerges as the primary catalyst for cognitive development. This finding corroborates Subotnik, Olszewski-Kubilius and Worrell’s (2011) notion that giftedness unfolds into talent through structured support. Enrichment opportunities such as Olympiad training not only deepen conceptual mastery but also contribute to long-term academic self-belief. Similarly, Gagné’s (2004) differentiation between natural aptitude and developed skills is vividly illustrated; gifted potential is actualised only when teachers provide appropriate challenges and recognise individual differences. In contrast, peer collaboration serves as a crucial social stimulant. Participants' descriptions of friendly competition and communal problem-solving areas align with Burney's (2008) focus on social context in education.

The results indicate that conventional mixed-ability classes may unintentionally overlook the psychosocial and cognitive needs of gifted learners, particularly those that use a standard equalising approach. Gifted learners in South Africa run the risk of social isolation and under-stimulation in overcrowded classes with little opportunity for differentiation. To better match learner preparedness and decrease boredom, South African educators may implement pull-out enrichment clubs, compressed curricular units, or tiered assignments, all of which are modelled after Singapore's differentiated teaching model.

Limitations of the study

This study's concentration on a single, well-resourced private school is one of its limitations. Gifted learners' experiences in public or under-resourced environments may vary significantly from private schools. Cross-context comparisons could be investigated in future studies to find effective practices in various learning contexts. Furthermore, longitudinal research monitoring shifts in self-perception and motivation over time might provide insight into how early treatments influence long-term results.

Conclusion

This study shows that when intrinsic desire, challenge that is adequately calibrated, and supportive social situations come together, mathematically gifted learners flourish. Peers spark group identification and collaborative refinement, while teachers are essential in organising advanced inquiry and confirming learners' self-efficacy. Teachers should use diversified assignments, structured peer-learning opportunities, and focused enrichment to maximise the potential of gifted learners, particularly in settings with limited resources and differentiation. By combining theory and empirical knowledge, this study emphasises how important it is to match formal education to the special cognitive and psychosocial requirements for learners who are mathematically gifted. This will support their academic success and their lifelong curiosity and self-assurance.

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FROM SYSTEMIC EVALUATION TO EFFECTIVE CLASSROOM PRACTICE IN MATHEMATICS EDUCATION IN SOUTH AFRICA: A LENS THROUGH TIMSS, SASE AND SEACMEQ

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ABSTRACT

This paper sheds light on three large scale systemic evaluations conducted recently, namely TIMSS, SASE and SEACMEQ V with a view to drawing comparisons and filtering through lessons that would be used to transform practice and policy-making in Mathematics Education.

The findings have critical implications for teaching practice. They underscore the urgent need for targeted interventions focused on addressing foundational literacy and numeracy gaps, particularly in early grades and for disadvantaged learners. Teachers require enhanced support and continuous professional development, specifically in pedagogical content knowledge and the ***integration of Assessment for Learning (AfL) practices****. AfL is highlighted as key to identifying and addressing specific learning needs and using assessment data formatively to guide instruction. The reports collectively provide diagnostic insights that can and should be used by teachers and school leaders to understand the specific barriers to learning in their contexts and inform tailored classroom strategies. Ultimately, the synthesis of findings from these reports offers a clear mandate for practice: moving beyond general approaches to implementing evidence-based, context-specific strategies aimed at ensuring all learners, especially those from marginalized backgrounds, receive the support needed to achieve required proficiency levels.*

Several recommendations made by the studies have been lifted up in this paper with crucial proposals for improving teaching and learning and the allied processes that support classroom practices. This paper argues that if data from systemic evaluations can be further analysed and repackaged for actions impacting curriculum, teachers and assessment both the equity and quality of Mathematics education can be substantially improved.

Introduction

Systemic evaluation in education is a strategic process that aims to assess the effectiveness and quality of education systems at scale, typically at the national and/or provincial level. In South Africa, systemic evaluation emerged as a pivotal mechanism in the post-apartheid era, providing empirical data to monitor and redress historical inequalities while informing educational planning and policy development (Department of Education [DoE], 2003; Spaull, 2013). This paper sheds light on three systemic evaluation reports, namely the South African Systemic Evaluation (SASE, 2022), the Trends in Mathematics and Science Study (TIMSS, 2023) and the Southern and Eastern Africa Consortium for Monitoring Evaluation Quality (SEACMEQ V, 2021) with a view to comparison and filtering through crucial lessons to be used in improving Mathematics policy and practice in the classroom.

Participation in these studies puts South Africa against the best and brave in the world. While South African performance in these tests is usually in the lower rung, the courage of the country to participate and to confront its performance is commendable. This paper explored how the studies were conducted, whether or not there were similar focal areas and how the recommendations could be used to improve Mathematics.

Background

The South African *consistent* participation in large-scale evaluation studies such as the Trends in International Mathematics and Science Study (TIMSS), the Southern Eastern Africa Consortium for Monitoring Educational Quality (SEACMEQ) dates back a few decades. Recently South Africa participated in TIMSS in 2011, 2015, 2019 and 2023; also the country participated in SEACMEQ II, III, IV and V.

TIMSS is a global and international studies involving over fifty countries. In the recent TIMSS (2023) fifty-nine countries participated, of which only three African countries including South Africa took part. On the other hand, SEACMEQ is a regional study made up of sixteen African member countries, of which nine (9) countries participated in the recent SEACMEQ V (2021). It should be noted that each time South Africa participates, all its nine provinces do participate affording comparison across provinces.

Further, after the Annual National Assessment (ANA) was discontinued in 2014, South Africa resumed the national systemic evaluation called South African Systemic Evaluation (SASE) in 2022. Both the ANA and the SASE were own systemic initiatives of a nation holding a mirror at itself in its quest to improve the equity and quality of its own education system.

While the international TIMSS (2023) tested Grade 4 and Grade 8 learners, South Africa participated with Grade 5 and Grade 9 in Mathematics and Science. SEACMQ V (2021) focussed on reading, Mathematics and HIV AIDS knowledge for Grade 6. The third study under comparison, the SASE (2022) focussed on reading and numeracy for Grades 3, 6 and 9. Indeed, the fact that South Africa participated in three consecutive years in similar studies made comparison of more educational sense.

For the purposes of this paper, however, the focus has been limited to Mathematics particularly in Grades 5,6 and 9.

Research Focus

The research focus for this paper was informed by the DBE intention of "unwavering commitment of the Department of Basic Education towards evidence-based decision-making that has currency and relevance" (SASE, 2022:77). This was echoed by SEACMEQ V (2021:126) of offering "policy and practice recommendations aimed at enhancing the quality and equity of education in South Africa". To sharpen this intention, the following research questions guided the comparison:

(i) What instruments were used in the three studies?
(ii) What findings and recommendations were arrived at?
(iii) How to use the recommendations to improve practice in the Mathematics classroom?

Literature Review

The introduction of the Systemic Evaluation (SE) program by the national Department of Education in 2001 marked a significant shift toward evidence-based educational governance (DoE, 2003). Since then the DBE participated in a range of large-scale evaluation internationally, regionally and nationally.

In addition to cognitive testing, a number of studies started to incorporate contextual instruments to examine inputs from factors surrounding the classroom, such as school management, family, socio-economic elements (Taylor et al, 2010, SEACMEQ III, IV and V, TIMSS (2023), SASE (2022)). These contextual factors began lifting how the socio-economic inequalities affected learning in the classroom.

While empirical evidence from these studies consistently showed South Africa's low levels of learner achievement, South Africa continues to value its participation in order to reflect on itself. With this, crucial pointers emerged that informed the need for differentiated interventions, and prioritized resource allocation (DBE, 2017).

Taylor (2019) noted the undesirable negative effect of "high-stakes nature of some assessments" (referring to ANAs) that encouraged teaching to the test and thereby distorting pedagogical practises. Kanjee & Sayed (2013) and TIMSS (2023) noted challenges related to test validity, curriculum alignment, and teacher feedback mechanisms. Fleish (2018) noted that 'effective use of data for pedagogical transformation remained uneven'.

This paper proposes closer analysis of the studies and responses in order to inform professional development, classroom teaching as well as assessment. Instead of the claim of "teaching to the test", the Mathematics community will be alive to these crucial studies and will be better placed to use them effectively to '*transform'* and improve pedagogy.

Methodology

With the intention to inform classroom practice, the three systemic reports: SEACMEQ (2021), SASE (2022) and the TIMMS (2023) were scrutinized. The scrutiny involved how the studies were conducted, the instruments used, the areas of focus, their findings and the recommendations they had drawn. The studies were juxtaposed for comparison so as to inform strategic choices in improving classroom practice.

The three research questions explored were:

- What instruments were used in the three studies?
- What findings and recommendations were arrived at?
- How to use the recommendations to improve practice in the Mathematics classroom?

The areas of focus, albeit referred to differently in the studies, broadly related to learner and teacher factors, gender issues, socio-economic factors and recommendations. Threads that pointed to possible actions that would improve the teaching and learning of Mathematics in the classroom were pulled out.

Findings and Observations

Casting a lens through these three large-scale assessment was crucial so as to see commonalities, and extract pointers for development and improvement. All three studies utilized a 0 -1000 scale to measure performance, with 500 as a centre point. A summary of the three studies is reflected in Table below.

Table 1: A Summary of the features of the three Studies: TIMSS, SASE and SEACMEQ

ATTRIBUTES	TIMSS	SASE	SEACMEQ
Participation	309 schools in at Grade 5 and 300 schools at Grade9	52 099 learners at Gr 6; 38 920 learners at Gr 9	6629 learners from 298 schools in 9 provinces 1 036 teachers
Focus of Study	Grades 5 & 9 Numeracy and reading	Grades 3, 6 &9 Mathematics and Reading	Grade 6 Mathematics, Reading and HIV Knowledge
Instruments' & Focus	Learner performance; Contextual factors of school Socio-economic factors of learners	Learner performance; Contextual factors at Micro-, Meso- and Macro-levels	Learner and teacher testing; General conditions of schooling and related factors such as LOLT and LTSM
Findings	A score of 397 at Gr 9 with 397 for boys and 401 for girls A score of 362 at Grade 5, with 348 for boys and 376 for girls At Grade 9: 60% of the learners performed at levels 1&2, with 45% of this at level 1. At Grade 5: 52% performed at levels 1&2, with 35% of these at level1.	At Grade 6, 64% perform emerging (16%) and evolving (48%) levels (equivalent to Cognitive levels 1 & 2) At Grade 9, similar performance is 60%. Emerging and evolving levels are 23% and 37% respectively. At Grade 9, 56% of girls and 67%of boys are at the emerging and	A score of 525 for learners and 759 for teachers 42% of learners performed at levels 1&2: 33,9% at level 2 56,1% boys and 38,4% of girls performed at levels 1&2. 92,5% of teachers performed at levels 3 &4, with 1.4 % teachers at level 1

		evolving performance levels.	
Recommendations	Produce a diagnostic report for classroom improvement Undertake a standard setting exercise to develop appropriate performance levels Expand capacity development for education officials to analyse, report and use results from large scale studies	Roll out extensive AFL teacher training; Use summative data for formative purposes (and vice versa) Strategic use of Teacher assistants (deployment, training and utilization)	Address equity and provincial disparities: resources and development; Promote gender-responsive pedagogy. Strengthen Foundational numeracy skills

Note: This paper focussed on Mathematics in Grades 5,6 and 9 in the studies. Micro level analysis include school, classroom and home contexts

Strategic Features Across the three studies

With a view to using empirical evidence from the three studies as a springboard from which informed development and improvement can happen, the following have been identified:

(i) Most learners are languishing in or captured at cognitive levels 1 and 2, which are described as “emerging” and “evolving”. This is observed across the three studies and it a picture which the country has to be intentional about changing, with a quest to move to level 3 (and even 4) of performance in the classroom and in the next systemic assessments.

(ii) Performance of boys compared to girls is consistently lagging behind in all the three studies. It is probable that this picture is the same in other summative assessments such as the NSC examinations. This calls for strategic and intentional programmes to assist the boy-learner, and begin to rethink initiatives such as the ‘girl learner camps’ and ‘take the girl-child to work’.

(iii) The need for rolling out the Assessment for Learning training for teachers and subject advisors.

(iv) Teacher knowledge, as shown in SEACMEQ V, cannot always be blamed. Teachers performed at levels 3 & 4 comfortably, while learners appeared ‘trapped’ at level 2. Indeed, other factors should be considered to explain the picture of Mathematics performance.

What follows is a proposal to engage these observations into concrete action steps that would lead to policy and classroom practice improvement.

Towards Effective Pedagogy in the Classroom

Large-scale systemic evaluation and allied forms of assessment should not end with reports but active and intentional effort should be taken to incorporate the recommendations in programmes for development and improvement. This section purports to pull the analyses from the three reports together into insights that will improve classroom practice.

(a) Professional development and teaching to enhance conceptual understanding

The three systemic reports are using competency or performance levels in their analysis of learner performance.

Table1: Performance levels related to Cognitive levels

TIMSS Grade 9	SASE Grades 6 &9	SEACMEQ Grade 6	CAPS ASSMT COGNITIVE LEVEL
Low Benchmark *Knowing* 45% perform level	Emerging 23% perform at this level	a. Pre-Numeracy b. Emergent Numeracy 9% perform at this level	1 Knowledge Elementary information and straight recalll
Intermediate Benchmark *Knowing* 15% perform at this level	Evolving 37% perform at this level	c. Basic Numeracy d Beginning Numeracy Most of learners (67.7%) perform at this level	2 Routine Procedures Performing well known procedures
High Benchmark *Applying*	Enhancing	e. Competent Numeracy f. Mathematically Skilled	3 Complex Procedures Complex calculations and higher order reasoning
Advanced Benchmark *Reasoning*	Extending	g. Concrete Problem Solving h. Abstract Problem Solving	4 Problem Solving Unseen, non-routing problems; No obvious route to the solution

The Table 1 maps the three systemic studies on the basis of the performance levels that each used, and these are further mapped to the CAPS cognitive levels. The SEACMEQ performance was explained in 8 performance levels which were then grouped in pairs so as to align with the other studies and allow comparison. In addition, alphabetic numbers as opposed to numeric were used the SEACMEQ so as to avoid possible confusion with the cognitive levels.

From the Table it can be observed that combining levels 1 and 2, learner performance is reflected as 60%, 60% and 76.7% for TIMSS, SASE and SEACMEQ respectively. The descriptors for these levels are indicated in the Table as "low benchmark", "emerging", emergent/beginning numeracy", etc. It is concerning that on a national, regional and global stage most of the learners are at the emerging or beginning understanding in Mathematics, somehow 'trapped at cognitive levels 1&2.

The proposal is to explore how performance in Mathematics can be decisively propelled towards the next cognitive level which enhances application of skilled and competent operations in Mathematics. The discourse on Cognitive level 3 is usually depicted when assessment is done with an attempt to 'set suitable assessment tasks' but it appears discourse becomes silent when actual teaching happens, and when professional development is planned and implemented.

This cognitive level is mapped to the SASE report (2022) as the *Enhancing level which is* characterized as

> Learners demonstrate the required grade-level understanding and skills, *apply their knowledge in authentic contexts*, and *are moving towards independent learning*. They benefit from moderate guidance and targeted support, *with tasks encouraging critical thinking, problem-solving, and real-world application. (page 13)* [Emphasis added]

In order to help the learning performance to move from the "emerging" and "evolving" levels towards the "enhancing level" (cognitive level 3) and probably higher, we propose that development and teaching should be guided by the Teaching Mathematics Framework (TMU, 2018). The TMU Framework proposes a multi-dimensional approach for teaching mathematics, aiming to transform teaching to achieve learning for understanding. The four dimensions are *conceptual understanding, procedural fluency, strategic competence, and reasoning*. These are interdependent, intertwined, and they should all be underpinned by a learning centred classroom. The latter can facilitate a dynamic interaction of the four dimensions with an environment where mathematical ideas can actively engaged leading to the attainment of "independent learning ... critical thinking, problem-solving, and real-world application" of the aspired Enhancing level.

(b) Active use of Assessment for Learning (AfL) in the classroom

UNESCO & DBE (2020) advocated for a more integrated and developmental approach to systemic evaluation. This includes the alignment of assessments with learning progressions, increased emphasis on **formative assessment practices**, and the strengthening of data literacy among educators and district officials. Furthermore, (Sayed & Ahmed, 2015) indicated calls to situate assessment within a broader ecosystem of accountability that supports teacher agency, rather than punitive compliance, and a growing consensus on the need to balance **national assessments** with **school-based assessments** that are contextually responsive and culturally sensitive.

The DBE (2012) described assessment for learning in the teaching and learning process in the classroom, as a crucial component of curriculum delivery. This involves gathering information about a learner's progress, interpreting that evidence, and using it to inform both teachers and learners about where they are in their learning, where they need to go. This is formative in nature. The World Bank Report (2018:17) says "formative assessment by teachers help guide instruction and tailoring teaching to the needs of students. Well prepared teachers do not need to operate in the dark: they need to know how to assess the learning of students regularly, formally and informally".

Knowing where students are, teachers can use assessment of suitably varied cognitive levels and continuous feedback in the classroom to achieve conceptual understanding. Differentiated strategies can help teachers set teach for conceptual understanding and set tasks that would help learners meaningfully operate at the "enhancing" and "extending" levels.

Pulling it Together: Recommendations

Continuing professional development (CPD) should be rethought, restructured to be responsive to the results of systemic evaluation. The programmes should incorporate the four cognitive levels from design to implementation so that teachers acquire skills to teach and to assess towards conceptual understanding. Differentiated approaches, with suitably broadened assessment tasks and requisite frequent feedback are aimed at moving from basics to enhancing and applying understanding to various problem situations in Mathematics.

Formative assessment broadly and assessment for learning in particular is recommended to be the vehicle for grounding the basics and expanding into cognitive level 3 performance with daily tasks in the classroom.

A balanced gender approach to curriculum delivery should be intentional in supporting the boy-child without neglecting the girl-child. The boy-child performance in Mathematics should be given increased attention to change the undesirable trend evident in most summative or systemic assessments.

Conclusion

The World Bank Report (2018) would conclude "Beyond being a basic human right, education – done right- improves social outcomes in many spheres of life" (p.27). The Report continues to advise on how this could be done: "Assess learning -to make it a serious goal, *Act on evidence* [our emphasis]– to make schools work, and Align actors – to make the whole system work for learning" (p.16). This paper argues that systemic assessment gives a picture that needs to be "acted on" as evidence towards development and improvement. Evidence from systemic evaluations should be further analysed and repackaged for actions impacting curriculum, teacher development and assessment so that both the equity and quality of Mathematics education can be substantially improved in South Africa.

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USING THE "NO OPT OUT" TEACHING STRATEGY TO IMPROVE LEARNER PARTICIPATION IN MATHEMATICS

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Abstract

This paper explores the application of the "No Opt Out" teaching strategy in the context of mathematics instruction and its potential to improve learner participation and understanding. Mathematics often presents a unique challenge for learners, especially in urban and rural areas, as it requires both conceptual understanding and procedural fluency. The "No Opt Out" strategy, which ensures that learners are involved in the learning process, is particularly effective in overcoming the barriers of hesitation and uncertainty that many learners face in mathematics. I chose to discuss the "No Opt Out "strategy because of its proven effectiveness in encouraging participation and fostering a growth mindset among learners. In this short paper, l will explain how the "No Opt Out "strategy works, share some examples from my mathematics classroom, and reflect on its benefits and its challenges.

Introduction

The "No Opt Out" strategy involves ensuring that every learner is engaged in answering a question, even if they initially cannot answer it correctly. The key idea is that the educator does not accept " I do not know " as an answer. Instead, the educator offers support or rephrases the question in a way that helps learners to arrive at the correct response. After assisting, the educator asks the same question again, allowing the learner to demonstrate understanding.

The "No opt out" strategy works particularly well in mathematics, where concepts often build on one another. A learner who is unsure about a particular problem can benefit from guided practice or hints before being asked to provide an answer, ensuring that the learning process is not interrupted. Examples one and two below show how the "No Opt Out " strategy can be used in a mathematics lesson.

Example 1: Step -by Step Problem solving

In this example, a learner is asked to solve a problem on the chalkboard. If the learner struggles to begin the problem, the educator offers hints to break the problem down into smaller steps. For example, if the learner is asked to solve for x in the following expression, 2x -8 =12, and if the learner does not know how to begin, the educator might prompt the learner with a question such as " What should we do first to isolate the variables?"

After providing a hint, the educator asks the same question again. Once the learner begins solving the problem with the educator's guidance, the learner can work through the steps and arrive at the correct answer. Afterward, the educator may ask a learner to explain his or her reasoning to the class, reinforcing the importance of understanding the steps involved.

Example:

Educator: "What is the first step in solving the equation 2x -8= 12?"

Learner: "I'm not sure."

Educator: "Let's start by adding 8 from both sides. What do we get?"

Learner: "Oh, 2x-8 + 8= 12 + 8."

Educator: "Great! Now, what's the next step?"

*Learne*r: " 2x = 20 "

Educator: Wonderful, what is the next step?

By using this strategy, learners gain confidence in their ability to approach and solve problems, knowing they will receive support if needed.

Example 2

In this activity, learners work in pairs or small groups to solve a challenging problem. If one learner is unable to contribute a solution, the educator will prompt the other members of the group to help or guide the learner through the problem. This ensures that the learner who had a challenge is not left out of the learning process. After the learner receives help, the educator asks him/ her the same question to check for understanding. This process also promotes peer to peer learning, which is a valuable component of the "No opt Out" strategy.

Educator: " let us solve the equation 3(x-10)= 2(2x-3). Can someone explain what the first step is?

Learner A: " I Do not know how to start'

Educator: What would happen if you expand the LHS?

Learner B : " We will get 3x -30 = 4x-6"

Educator : Exactly , Now Learner A try to solve the problem from that step.

By encouraging peer collaboration, learners feel supported, and the learning process becomes less intimidating.

Conclusion

The " No Opt Out" teaching strategy is an invaluable tool for fostering learner participation in mathematics. It ensures that all learners are actively involved in the lesson, even when they may initially struggle with a concept. Through guided assistance and repeated opportunities to answer questions , learners gain confidence in their abilities and deepen their understanding of mathematical concepts. my experience with this strategy has been overwhelmingly positive, as it encourages a growth mindset and ensures that no learners feels excluded from the learning process. However, it is important to balance support with opportunities for independent thinking so that learners are not evenly dependent on prompts. When implemented thoughtfully, the " No Opt Out" strategy significantly enhances learners participation and achievement in mathematics.

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THE ROLE OF MATHEMATICS DEPARTMENTAL HEADS IN RESOURCE MANAGEMENT: ADDRESSING INEQUITIES IN SOUTH AFRICAN SECONDARY SCHOOLS

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Abstract

This paper examines the role of Mathematics Departmental Heads (MDHs) in managing both print and digital resources to mitigate these inequities. It outlines how effective resource management by MDHs encompassing the selection, distribution, and utilisation of materials can improve teaching and learning outcomes in under-resourced contexts. Key challenges are identified, including disparities in resource allocation between schools, insufficient teacher training, and the dual burden on MDHs as both teachers and administrators. The discussion highlights relevant frameworks and policies such as the Curriculum and Assessment Policy Statement (CAPS) and the Department of Basic Education's ICT integration initiatives, illustrating their impact on resource management practices. Case examples and models from current literature (including insights from Kgosana's study and others) are incorporated to demonstrate successful strategies, like collaborative planning and the integration of technology aligned with the fourth industrial revolution. Through an academic analysis supported by evidence, the paper argues that strengthening the instructional leadership capacity of MDHs and ensuring equitable resource distribution are essential steps toward improving mathematics achievement in South African secondary schools. It concludes with recommendations for empowering departmental heads through professional development, policy support, and community collaboration to create a more equitable and effective mathematics education environment.

Introduction

Mathematics education in South Africa is shaped by enduring inequalities that mirror the country's broader socio-economic divides. Despite sustained investment in the education sector, many secondary schools, particularly those in historically disadvantaged communities, face persistent shortages in quality resources, leading to poor learner performance (Spaull, 2019). National and international assessments confirm the severity of the problem: only 37% of Grade 5 learners demonstrated basic mathematical competence, and South Africa ranked last among 39 countries in the 2019 TIMSS study (Mullis et al., 2020). These outcomes reflect a "silent crisis" in foundational numeracy, linked closely to the inequitable distribution of educational resources (Van der Berg, 2015).

While the Curriculum and Assessment Policy Statements (CAPS) set uniform learning standards, implementation is inconsistent across schools due to disparities in access to resources (Department of Basic Education [DBE], 2011). Affluent schools benefit from up-to-date materials and digital tools, while less-resourced schools often struggle with outdated textbooks and overcrowded classrooms (Spaull & Jansen, 2019). Within this landscape, Mathematics Departmental Heads (MDHs) play a pivotal role. As experienced educators with leadership responsibilities, MDHs operate as instructional leaders and resource managers within school management teams. They oversee curriculum alignment, monitor teacher performance, and are tasked with securing and managing teaching resources. If empowered and supported, MDHs can address inequities by lobbying for resources, facilitating optimal use of available materials, and guiding technology integration (Kgosana, 2024).

With the rise of the Fourth Industrial Revolution (4IR), schools must now navigate the integration of digital technologies in education. Tools such as interactive software, online platforms, and digital libraries have the potential to enhance mathematics instruction, especially in under-resourced settings (OECD, 2021). South African policies like the e-Education White Paper, Thutong portal, and Digital Classroom Project support this transition (DBE, 2004; Vodacom Foundation, 2020). However, effective implementation depends on school-level leadership, particularly the MDHs, who must develop new capacities to manage both traditional and digital resources effectively.

This paper explores how MDHs in South African secondary schools can be transformative agents in addressing inequalities through strategic resource management. It examines existing disparities, outlines the roles and challenges faced by MDHs, and proposes equity-driven strategies such as aligning resources to CAPS, using Open Educational Resources (OER), and fostering teacher collaboration. Drawing on recent South African educational literature and policy, including Kgosana’s (2024) study on MDHs, the paper concludes with actionable recommendations to strengthen MDHs' capacity to ensure all learners benefit from quality mathematics education.

Theoretical Framework

Instructional leadership theory positions departmental heads as mediators of curriculum and practice, influencing teacher efficacy, curriculum fidelity, and resource allocation. The model asserts that academic performance improves when subject leaders are involved in guiding teaching and learning processes. Variation Theory complements this by offering a lens through which to structure the selection and use of teaching materials. By varying examples and resource types, teachers can make abstract concepts more accessible. For MDHs, understanding these variations can guide the selection of textbooks, manipulatives, and digital tools.

The study is underpinned by instructional leadership theory and Variation Theory. Instructional leadership highlights the influence of department heads in supporting teaching and learning through effective curriculum implementation and teacher support. Variation Theory (Marton, 2015; Pang & Marton, 2003) emphasizes the role of exposing learners to multiple forms of a concept to deepen understanding. These frameworks are applied to explore how MDHs can lead strategic and inclusive resource use in the mathematics department.

Review of Related Literature
Additional research by Bush et al. (2010) argues that middle leaders, particularly subject heads, play a critical role in improving instructional practices. They act as translators of policy and innovation into subject-specific practice, especially when empowered with professional development and autonomy. Furthermore, Bektaş et al. (2022) demonstrated that distributed leadership and collaborative decision-making can mitigate challenges in under-resourced contexts, suggesting that MDHs who share leadership roles with teachers are more effective in resource utilisation.

South Africa's education system continues to reflect stark inequalities, despite policies aimed at redress (Spaull, 2013; Sayed & Motala, 2012). Research shows that resource disparities are a major contributor to learner performance gaps (Donohue & Bornman, 2014; van Niekerk & Blignaut, 2014). MDHs are seen as key to bridging this gap due to their dual role as instructional leaders and administrators (Bush & Glover, 2016). However, their effectiveness is often undermined by systemic constraints including limited training, workload, and insufficient autonomy (Malloy, 2017; Ngema & Lekhetho, 2019).

Methodology
Data collection included both oral and written interviews, allowing participants to reflect deeply on their experiences with minimal influence from researcher presence. Interview questions focused on key areas: resource acquisition processes, decision-making roles, collaboration with colleagues, and specific case studies of resource gaps or innovations. Participants were anonymised and transcripts were coded to identify themes using Microsoft teams, ensuring methodological rigour and ethical integrity.

This qualitative study used semi-structured interviews and document analysis involving Mathematics Departmental Heads from Johannesburg North district schools. The participants represented a mix of public no-fee and fee-paying schools. Data were analysed thematically, focusing on resource-related practices, leadership experiences, and systemic constraints. The qualitative approach was chosen to capture the lived realities and contextual nuances of MDH leadership.

Findings and discussion
Additional findings show that some MDHs implemented departmental resource audits and catalogues to track availability and ensure fair distribution. Others created their own supplementary materials when DBE-issued textbooks were insufficient or misaligned with CAPS requirements. Some MDHs also reported innovative use of mobile technology (such as WhatsApp groups) to disseminate revision materials and facilitate after-school tutoring in communities with limited infrastructure. These case studies illustrate not only the resilience but the creative leadership of MDHs when given support and recognition.

Findings reveal several core themes: (1) MDHs are often burdened with dual teaching and administrative roles, limiting their time for leadership tasks; (2) Professional development for MDHs is inconsistent and poorly aligned with their specific responsibilities; (3) Decision-making is constrained by school governance and budget limitations, especially in no-fee schools; (4) There are notable innovations among some MDHs, including resource-sharing, use of open digital platforms like Siyavula, and strategic collaboration with teachers. These findings reinforce that MDHs can be powerful agents for change but require targeted support and structured empowerment frameworks.

Implications for Mathematics teaching, learning and further research

For teaching, the implication is that resource use must be intentional and varied, aligning with both curriculum objectives and learner diversity. MDHs must lead initiatives that identify gaps in content delivery and intervene by sourcing or developing appropriate teaching aids. For learning, equitable access to manipulatives, digital platforms, and exam preparation materials can significantly enhance learner engagement and success. Monitoring how these resources are used and gathering learner feedback, should become a core MDH function.

Future research should examine comparative models from other countries, particularly in the Global South, where similar roles exist. Exploring cross-country professional learning communities may further enhance MDH capacity building. Strengthening the leadership role of MDHs could improve resource equity and mathematics learning outcomes. Key recommendations include formalising MDH roles in policy; providing tailored professional development in resource planning and digital integration; and promoting communities of practice. Further research should explore longitudinal effects of MDH empowerment and the impact of resource strategies on learner achievement.

Focus of the study and research questions

The study seeks to understand whether empowering MDHs can bridge the persistent gap in mathematics achievement between affluent and under-resourced schools. By exploring real school-based scenarios, this research contributes new evidence to the national conversation on educational equity.

The research also aims to respond to recent national calls, including the National Development Plan 2030, which urges better resource allocation and leadership in schools. MDHs are uniquely situated to influence how policy translates into classroom practice, especially in mathematics, a subject central to science, technology, engineering and mathematics (STEM) fields.

This research investigates the pivotal role that Mathematics Departmental Heads (MDHs) play in managing resources in South African secondary schools. The focus is on how MDHs can address resource inequities that negatively impact mathematics performance, particularly in under-resourced schools. The primary research questions are: How do MDHs influence the availability and use of mathematics teaching resources? What challenges and successes do they encounter in promoting equitable resource management? The significance of this study lies in its potential to inform policy and practice aimed at narrowing educational inequalities through school-level leadership.

How can school principals be systematically trained to empower MDHs as collaborative decision-makers?

What sustainable funding models can ensure equitable digital tool access in no-fee schools?

How might an MDH-led resource audit system be standardized nationally for policy planning?

Resource inequities in South African Secondary Schools

Educational inequality in South Africa is deeply rooted in historical and socio-economic disparities, with resource distribution in schools reflecting and perpetuating this inequity (Spaull, 2013). Although the quintile funding model aims to prioritise no-fee schools (Quintiles 1–3), practical challenges remain. Poorer schools often lack basic mathematics resources such as textbooks, manipulatives, and updated materials aligned with the CAPS curriculum, while affluent schools are equipped with modern tools, digital devices, and enriched learning environments (Spaull & Jansen, 2019; DBE, 2011).

This disparity leads to vastly different learning experiences and outcomes. Learners in under-resourced schools often do not reach basic competence levels in mathematics, whereas those in well-resourced schools excel, with some performing at international benchmarks (Mullis et al., 2020). Socio-economic conditions exacerbate the divide: wealthier communities can supplement state support, while impoverished ones depend entirely on inconsistent government provisioning (Van der Berg, 2015).

Beyond tangible resources, the inequity includes unequal access to enrichment opportunities like exam preparation materials, maths clubs, and digital tools. The COVID-19 pandemic highlighted the digital divide; schools with ICT infrastructure adapted quickly to online teaching, while others experienced prolonged disruptions (OECD, 2021). Although policies like the DBE's ICT in Education strategy and the School Connectivity Project aim to address this, implementation remains uneven, particularly in rural areas (DBE, 2004; Vodacom Foundation, 2020).

Effective resource use also depends on school-level leadership. In some schools, proactive management maximises limited resources creatively; in others, poor administration leads to underutilised or mismanaged materials. This makes the role of Mathematics Departmental Heads (MDHs) critical. MDHs can drive equitable use of resources within their departments, but they too face limitations that need to be addressed through training and support (Kgosana, 2024). Inequitable access to quality mathematics resources, traditional and digital, remains a key barrier to educational equity in South Africa. Addressing it requires not only increased provisioning but strategic, accountable resource management led by capable school leadership.

The Role of Mathematics Departmental Heads in Resource Management

A core responsibility of MDHs is to maintain alignment with curriculum requirements. Since mathematics is a high-stakes subject (crucial for higher education entry and tied to national development goals), CAPS provides detailed guidelines on content coverage for each term. MDHs must verify that the textbooks and supplementary materials used in their school adequately cover these topics at the appropriate cognitive levels. In South Africa, where curriculum changes have occurred (e.g., the transition from the old syllabus to CAPS in the early 2010s), departmental heads often play a role in evaluating new textbooks for adoption. They might lead a review process with their teachers to select a textbook from the list of DBE-approved materials that best suits their learners' context.

Additionally, they compile departmental resource inventories and identify needs: for instance, an MDH might realise the department is short of math sets for geometry and request funding in the school budget to purchase more or decide that an online tool like Siyavula (an open online math practice platform) would benefit learners and thus initiate its use. Essentially, MDHs must be both managers and innovators, managing day-to-day resource needs while also seeking out new resources or methods to enhance learning. Research has highlighted that this role is pivotal; effective leadership at the department level correlates with better student achievement, because it translates broad school policies into subject-specific actions that directly reach learners (Bush et al., 2010; Hallinger, 2018). For example, Hallinger (2018) emphasizes the importance of context in school leadership, a mathematics HOD must understand the specific challenges of teaching math in their school's context (be it large class sizes, language barriers, or otherwise) and manage resources accordingly to meet those challenges.

In managing resources, MDHs also must consider equity within the school. Larger secondary schools often have multiple mathematics classes per grade, sometimes even streams that are organised by ability. A departmental head should ensure that all these classes receive comparable resources. If one Grade 9 class has access to a set of mathematical instruments or a digital projector and another class does not, it can create internal disparities. To prevent this, MDHs might implement rotation systems (e.g., shared use of a single projector among classes) or advocate for additional units so each class is equally served.

Moreover, MDHs can target resources to where they are most needed: for instance, channelling extra revision booklets or tutoring resources to classes or learners that are struggling, thereby using resources strategically to uplift weaker students. In some cases, MDHs coordinate common exams or assessments for all classes in a grade, which is a practice that not only standardizes evaluation but also indirectly forces a standardisation of what content and resources have been covered. If one teacher has not been able to teach a section due to resource lack, it will come out in such processes, allowing the MDH to intervene (perhaps by lending materials from another class or arranging peer support between teachers).

However, fulfilling this multifaceted role is not straightforward, and many MDHs operate under significant constraints. They are usually also full-time teachers who carry their own teaching loads, which means the time they can devote to management duties is limited. A study by Malloy (2017) observes that departmental heads often find their focus "shifted away from instructional leadership to administrative responsibilities" due to workload. The next section delves into these challenges, acknowledging that while MDHs have the mandate to manage resources and support teachers, various barriers may impede their effectiveness. Understanding these challenges is essential for designing interventions that truly empower MDHs as champions of equitable resource distribution.

Challenges facing departmental heads in resource management

Despite their critical role, Mathematics Departmental Heads frequently encounter obstacles that hinder their ability to manage resources optimally and address inequities. These challenges stem from systemic issues, school-level conditions, and personal capacity gaps. Recognising and addressing these barriers is key to enabling MDHs to fulfil their potential as instructional leaders.

Administrative and teaching workload

One of the most immediate challenges is the sheer scope of responsibilities that MDHs shoulder. In addition to managing the math department, they typically teach several classes a day, prepare lessons and assessments, and handle routine administrative paperwork. The dual role of being both a teacher and a middle manager means MDHs are often stretched thin. Time that ideally would be spent on strategic tasks – such as analysing resource needs, coaching teachers, or researching new teaching tools, is often eaten up by urgent day-to-day demands like covering classes for absent teachers or compiling reports for the principal.

Malloy (2017) highlights that this juggling act can shift an HOD's attention away from instructional leadership towards administrative chores. Many MDHs report that they conduct departmental activities (like resource inventory updates or collaborative planning meetings) during their personal time, indicating that the formal school schedule often does not allocate space for these duties. This time poverty undermines thoughtful resource management; for example, an MDH might default to reusing the same textbook each year without reviewing whether a better option exists, simply because there is no time to explore alternatives. It also contributes to burnout and diminishes the enthusiasm needed to drive innovation in the department.

Insufficient professional development

Effective resource management, especially in a rapidly changing educational landscape, requires specific skills and knowledge, from evaluating the quality of a textbook or app, to budgeting and procurement processes, to understanding how to integrate a new tool into pedagogy. However, many MDHs have not received formal training in these areas. They often advance to the HOD position due to being strong classroom teachers or experienced staff, not necessarily because they possess management qualifications. Donohue and Bornman (2014) noted in an inclusion context that teachers in South Africa generally lacked preparation for handling diverse needs; similarly, MDHs may lack preparation for their expanded leadership role.

The professional development opportunities that do exist (such as workshops run by provincial education departments or online courses) are sometimes generic and not tailored to the unique role of departmental heads. A recent comprehensive study in the African context identifies that *"insufficient professional development"* is a key challenge for MDHs in resource management. For instance, if a new mathematics software is rolled out by the DBE, the training might be provided to one or two teachers per school (often not the HOD), leaving the HOD to learn second-hand or not at all. This gap means that MDHs might not be fully aware of contemporary resources available or how to use them effectively. It also affects their confidence, an MDH who isn't comfortable with technology, for example, may be hesitant to advocate for digital resources or to guide their teachers on using them, effectively causing the school to underutilise potentially valuable tools.

Limitations in decision-making power and support

While MDHs are responsible for managing resources, they do not always have the authority or support to make significant changes. School resources are typically governed by the principal and the School Management Team as a whole, and budgets are approved by school governing bodies or district offices. As such, an MDH might identify a need (say, graphing calculators for the Grade 11 and 12 classes, or an extra mathematics teacher to reduce class size) but lack the decision-making power to secure those resources. If the school leadership does not prioritize that need, the MDH's hands are tied. Moreover, some MDHs operate in schools where leadership and governance are weak or fraught with challenges.

In extreme cases, issues like mismanagement or corruption can impede resource provision, e.g., funds earmarked for textbooks might be misallocated. Ngema and Lekhetho (2019) argue that weak management and bureaucratic inefficiencies in schools render many well-intended policies ineffective. In addition, inadequate support from the broader School Management Team is a recurring complaint. The literature and Kgosana's findings note that many MDHs feel they do not receive sufficient backing from principals or district officials when it comes to resource management. This could mean lack of guidance, delayed approvals for procurement, or insufficient involvement in planning processes. Without strong support, MDHs may be unable to implement changes that could address inequities, such as initiating a resource-sharing program with a neighbouring school or securing donations from alumni.

Gaps in content and pedagogical mastery among teachers

While not a challenge of the MDH directly, the capacity of the teachers in the department affects resource usage. South African studies have revealed that many teachers struggle with mathematics content knowledge and modern teaching strategies. As MDH, one must manage resources cognisant of this reality: providing advanced textbooks or complex digital tools to teachers who are not comfortable with them could lead to those resources being ignored or misused. Thus, an HOD often must first uplift the teachers' skills (through internal workshops or mentoring) before a resource can achieve its intended impact. This mentoring role is demanding and requires interpersonal skills and patience, especially if some teachers are resistant to new methods. In schools where an MDH is relatively young or new and must lead older teachers, there can be resistance to their directives on resource usage (we have always taught without fancy software, why change now? mentality). Overcoming such attitudinal barriers is a subtle challenge that requires the MDH to demonstrate the value of resources and possibly to enlist early adopters to showcase success.

External inequities and constraints

Departmental heads in historically disadvantaged schools face external challenges that their counterparts in better-resourced schools might not. These include larger class sizes (making resource distribution within a class harder, e.g., not enough textbooks for 60 learners in one classroom), language barriers (many learners learn in a second language, which means needing bilingual resources), and socio-economic issues (learners who cannot afford calculators or data for online tools at home). An MDH in a township school, for instance, might have to plan around the reality that students cannot take textbooks home (for fear of them being lost or damaged), which means ensuring sufficient class time for using the book. Or they might need to arrange fundraising for basics like math kits (rulers, protractors) because students don't have them. These efforts can be exhausting and may still not fully bridge the gap. It underscores that MDHs, while influential, operate within a larger system that needs to address fundamental resource allocation inequalities.

Strategies for equitable resource management by MDHs
Overcoming the challenges described above and leveraging the full potential of Mathematics Departmental Heads in resource management require deliberate strategies at both the school and system level. Below, we outline several approaches, drawn from research, policy, and case examples, that can empower MDHs to address resource inequities and improve mathematics teaching and learning outcomes.

Targeted professional development and communities of practice
To address gaps in training, professional development (PD) for MDHs should be tailored to their dual role as teachers and managers. This could include workshops or courses focused specifically on resource management skills: for instance, training on how to conduct a needs assessment for resources, how to evaluate digital learning tools, or how to align resources with CAPS effectively. Given the emphasis on technology integration, PD should also cover ICT competencies and pedagogical strategies for using digital resources, aligning with the Technological Pedagogical Content Knowledge (TPACK) framework which melds tech know-how with teaching practice (Mishra & Koehler, 2006). In South Africa, some initiatives have begun to recognise this need.

The Department of Basic Education's 2018 Professional Development Framework for Digital Learning encourages upskilling teachers and school leaders in ICT integration, outlining progression stages from basic digital literacy to innovative classroom practice. Embedding MDH-specific modules into such frameworks could yield benefits. Additionally, forming communities of practice can help. MDHs from different schools (within a district, for example) could meet periodically to share experiences, challenges, and solutions related to resource management. These networks, sometimes fostered by district offices or education NGOs, allow for peer learning. If one HOD has found a creative way to acquire low-cost math kits or has successfully implemented an online homework system, they can share the model with others.

Such collaboration not only disseminates good practices but also provides moral support, reducing the isolation an MDH might feel in a struggling school. Research by Bektaş, Ekiz, and Yildirim (2022) on distributed leadership in resource-constrained schools found that when leadership responsibilities (like resource planning) were shared among staff and supported through teamwork, schools were more resilient and adaptive. In practice, an MDH can foster distributed leadership by empowering math teachers to take lead on certain resource-related tasks (one might head up the math club or Olympiad training, another manages the math lab materials, etc.), thereby distributing the load and tapping into each teacher's strengths.

Strategic alignment with policy and leveraging government programs
Departmental heads should be well-versed in current educational policies and initiatives that can be leveraged to improve resources. For example, CAPS not only dictates *what* to teach but also gives guidance on the *kinds* of resources recommended (e.g., CAPS documents often list

suggested resources or experiments for certain topics). By closely aligning resource procurement with CAPS guidelines, MDHs ensure that materials directly support mandated curriculum outcomes. This could mean prioritizing the purchase of mathematical instruments, science kits, or specific manipulatives mentioned in the curriculum for certain grades.

Furthermore, the South African government and provincial departments regularly roll out programs aimed at equity, such as the distribution of Dinaledi maths and science grant resources to targeted schools, or the provisioning of ICT equipment through the ICT in Schools initiative. MDHs should actively engage with these programs: for instance, applying for inclusion if their school qualifies, or providing feedback on resource needs to education officials. Knowing the DBE's resource provisioning schedule (like when new textbooks are due for a curriculum cycle, or how often workbooks are delivered) allows an MDH to plan supplementary needs accordingly. A concrete example can be seen with the DBE's rollout of mathematics workbooks nationwide; these are standardised exercise books provided annually.

An MDH can build the department's assessment plan around these workbooks, ensuring every learner uses them fully (since they are free resources) and identifying where additional practice material might be needed beyond what the workbook covers. Additionally, South Africa's National Development Plan 2030 places strong emphasis on improving education quality and equity. One of its targets is to increase the number of learners achieving excellence in mathematics and to ensure that all schools meet minimum resource standards. MDHs can use the NDP's goals as a supportive reference when advocating for resources, framing requests as part of meeting national targets gives them added weight. For example, an HOD writing a motivation for a school to get a computer lab for math and science might cite the NDP and DBE's e-Education policy to argue that such infrastructure is not a luxury but a necessity for meeting the country's educational vision.

Open educational resources and partnerships

In scenarios where budgets are tight, MDHs can look beyond traditional channels and tap into open educational resources (OERs) and partnerships. South Africa has been a hub for some innovative OER in mathematics. Maskiverse and Siyavula project, for instance, provides free online mathematics textbooks and practice exercises aligned to CAPS for grades 8-12. An MDH can facilitate the use of Siyavula by ensuring the computer lab (if available) is open for learners to practice, or by downloading and printing exercises for classes that lack internet. Partnerships can also be local: collaborating with nearby schools to share resources or expertise. If a neighbouring school has a strong math department or specific equipment (like a math-focused computer lab), the MDHs could arrange inter-school resource sharing or joint learner workshops.

Corporate or community partnerships are another avenue, companies sometimes donate calculators or funds for science equipment as part of corporate social responsibility; an MDH can take the initiative to approach such companies with a clear proposal of what the department needs and how it will impact learners. Non-profits and universities might run outreach programs (for example, some universities host mathematics competitions or Saturday schools). By engaging with these, MDHs expand the resource base available to their learners beyond what the school alone can provide. The key is proactiveness and networking: an MDH who stays informed about opportunities (through professional associations, social media, or the DBE's circulars) can secure additional resources that directly benefit learners.

Integrating digital resources with pedagogical purpose

When it comes to technology, the mantra should be "pedagogy first, technology second." Departmental heads can guide their teams to integrate digital tools in ways that directly address learning challenges. For example, if learners struggle with visualizing geometric objects, the MDH might introduce dynamic geometry software like GeoGebra in lessons. One case recounted by a South African MDH involved teaching the trigonometric identity $sin\,x = cos\,x$ by having students graph both functions using software; visually observing the intersection of graphs helped students grasp the solution better than algebraic manipulation alone. This example illustrates how digital resources, when thoughtfully employed, can cater to diverse learning styles, some learners benefited from the visual representation. MDHs should encourage their teachers to identify pain points in the curriculum where learners typically falter and then consider what digital or hands-on resource could help. It could be simulation software for statistics, video tutorials for algebra, or even something as simple as an online quiz for immediate feedback.

Importantly, the MDH can coordinate a consistent approach so that all classes benefit: for instance, scheduling the use of a limited number of tablets across classes or arranging an after-school session in the computer lab for multiple classes to use a tool. The departmental head also has a role in setting norms and providing support for technology use, for example, deciding that all teachers will post supplementary materials on the school's online platform (if one exists) so learners can access extra practice, and then helping teachers learn to do so. In schools that lack much digital infrastructure, MDHs can start small, perhaps with widely accessible tools like the WhatsApp messaging platform to send out problem-of-the-day challenges or solutions to homework (keeping privacy and policies in mind). Each incremental step in integrating technology should be monitored for effectiveness, something an MDH can do by gathering feedback from both teachers and students. In this way, technology becomes a means to an end (better understanding, more engagement), rather than an end.

Strengthening internal departmental processes

Organisational strategies within the math department can make resource management more systematic and equitable. One idea is to develop a departmental resource management plan at the start of each year (or each term). This plan, led by the MDH but ideally co-created with the math teachers, would map out what resources are needed for which topics and when. For example, if Term 2 for Grade 10 involves a statistics project, the plan would note the need for graph paper, possibly access to a spreadsheet program, etc., by a certain date, so the MDH can arrange those ahead of time. The plan would also assign responsibility, maybe one teacher oversees maintaining the geometry instrument kits, another manages the digital content subscriptions, and so on. By distributing tasks, it not only lightens the MDH's load (reflecting a distributed leadership model) but also creates a sense of collective ownership of resources.

Regular check-in meetings (say, monthly department meetings) can include a standing agenda item on resource status: Are all classes set for the upcoming topic's materials? Any issues with textbooks or devices? This proactive stance helps avoid last-minute crises where a teacher suddenly runs out of a needed item or discovers half the calculators are broken right before an exam. Additionally, MDHs can implement simple but effective tracking systems, inventories for physical resources and usage logs for digital tools, to monitor how resources are being used and ensure nothing lies idle or gets lost without replacement. These internal processes, while perhaps mundane, create resilience in resource management. They also generate data that the MDH can use when advocating for more support; for example, showing the principal or district officials, a log indicating that the single computer in the math lab is used 40 hours a week by hundreds of learners can strengthen the case for funding more computers.

By adopting these strategies, Mathematics Departmental Heads can make significant strides in levelling the educational playing field. Many of these approaches hinge on the MDH taking initiative and being a proactive leader, which underscores why investing in the leadership development of HODs is so crucial. When MDHs are equipped with knowledge, supported by policy and peers, and given some autonomy to innovate, they become powerful agents of change. In turn, their departments become more collaborative and resource-rich environments, even within the constraints of broader systemic challenges. The final section of this paper will consider how policymakers and school leaders can further support MDHs, and it will summarise the key recommendations for ensuring that resource management by departmental heads indeed translates into reduced inequities and improved mathematics outcomes.

Policy implications and support frameworks

The Department of Basic Education has put in place various policies aimed at resource provision and ICT integration. For example, the Curriculum and Assessment Policy Statement (CAPS) not only standardizes curriculum content but as noted earlier, implicitly requires that every learner have access to certain resources (like textbooks and calculators) to meet the curriculum standards. Ensuring CAPS implementation goes hand in hand with ensuring resource availability. Education authorities should thus monitor resource distribution as part of curriculum monitoring, for instance, district curriculum advisors could include a resource checklist during school visits, verifying that schools (and specifically math departments) have the necessary materials for the topics in that term. If gaps are found, a remediation process (perhaps emergency provisioning or redistribution of surplus from other schools) could be triggered. Additionally, the DBE's White Paper on e-Education (2004) and the more recent ICT in Education Policy emphasise closing the digital divide in schools.

Translating these into action means allocating budget for devices, connectivity, and digital content, particularly targeting under-resourced schools. Policy could formalise the role of MDHs in this digital rollout: for example, involving them in training for new ICT equipment, or having them report on usage statistics to ensure accountability for provided tech resources. In Gauteng province, the "smart classroom" initiative equipped many schools with tablets and interactive boards; lessons learned indicate that without HODs championing their use, some devices gathered dust. Therefore, making MDHs key stakeholders in such initiatives (with possibly incentives or recognition for those who successfully integrate technology) can bridge the gap between policy intentions and classroom reality.

Frameworks for departmental leadership and accountability
Strengthening the role of departmental heads may also require formalizing their responsibilities and expectations. The Personnel Administrative Measures (PAM) in South Africa outlines duties of educators at various post levels. For HODs, it includes managing the department, but this can be elaborated to explicitly include resource management and equity responsibilities. If, for instance, HODs were required to submit an annual departmental resource report (detailing what is available, what is needed, and plans to address shortages), it would ensure the issue remains on the radar. Moreover, such documentation can feed into larger plans like the school's improvement plan or the district's resource allocation strategy. Another framework to consider is instructional leadership development programs.

Currently, much attention is given to training school principals, but similar efforts could target HODs. Universities or training colleges could offer certificate programs in middle management for schools, covering leadership, mentorship, and resource management. Not only would this build capacity, but it also professionalises the role, giving HODs a clearer identity and voice. Internationally, some education systems have created middle leader training recognizing that these individuals (like department heads) are pivotal in subject-specific improvement. Adopting such frameworks in South Africa would align with global best practices and reinforce the idea that MDHs are not just senior teachers, but leaders entrusted with achieving departmental excellence and equity.

Supporting equity-focused initiatives
To directly tackle inequities, policies could incentivise or mandate certain equity-focused actions. For example, a policy might require that all schools undertake an equity audit of resources annually, examining disparities in their own context (like between subjects, or between grades, or between learner groups). If a school finds that, say, Grade 12 math and science classes get priority in resources at the expense of Grade 8-9, the SMT and HODs could then plan to balance that. On a broader scale, the government's funding mechanisms could be sharpened. While the quintile system and no-fee policies channel more funds to poorer schools, within those schools perhaps additional earmarked funds for math and science resources could be allocated, acknowledging that these subjects often require more equipment. Some provinces have implemented a "maths and sciences grant" for this purpose, ensuring that a portion of school funding is strictly used for relevant resources (lab equipment, math kits, etc.) and monitored. MDHs, being closest to the subject needs, should have a say in how those funds are used. This implies that school principals and governing bodies need to include HODs in budgeting conversations. A supportive policy from DBE could encourage or require that inclusion.

Leadership and incentives
Finally, creating a culture that values and recognises the work of MDHs can motivate them to excel in resource management. Incentives could range from simple recognition awards (best performing department, most improved math results, etc.) to career progression opportunities.

If HODs see that effectively managing their department and improving results opens doors for them (like eligibility for specialist roles, or fast-tracking into vice-principal positions), they may be more inclined to invest effort into innovative practices. On the leadership front, principals play a key role: a principal who delegates authority, provides trust and backing to the MDH, and allocates time for departmental planning will enable that MDH to shine. Conversely, a micromanaging or indifferent principal can stifle the HOD's initiatives. Training for principals should therefore also emphasise how to empower and work with departmental heads as a team driving school improvement.

In summary, policies and frameworks at the national and school level should converge to empower mathematics departmental heads. By clarifying their role, providing necessary support and resources, and holding schools accountable for equitable resource distribution, the education system can ensure that every math department, regardless of the school's socio-economic status, has the tools needed to deliver quality education. When MDHs are supported by such an enabling environment, their individual efforts multiply into systemic change, narrowing the equity gaps that have long plagued South African education.

Conclusion

The pursuit of equitable mathematics education in South African secondary schools hinges on a multitude of factors, but this analysis underscores the central role of Mathematics Departmental Heads as catalysts for improvement. Tasked with managing both the human and material resources of their departments, MDHs operate at the nexus of policy and practice – translating broad educational goals into concrete classroom realities. By acknowledging their influence and bolstering their capacity, schools and policymakers can make significant strides in addressing the resource-based inequities that contribute to unequal learner outcomes.

In conclusion, strengthening the role of Mathematics Departmental Heads in resource management is not a silver bullet, but it is a highly leveraged intervention for addressing inequities in South African secondary schools. By equipping MDHs with the necessary skills, authority, and support, we empower them to turn resources (however limited) into real opportunities for learning. This approach aligns with the broader educational imperatives of South Africa – from the National Development Plan's vision of quality education for all to the immediate need for better results in national and international assessments. As schools implement the strategies and recommendations discussed, it is hoped that mathematics classrooms will increasingly become sites of equity, where with the guidance of dedicated departmental heads, every learner gains access to the tools and teaching they need to succeed in mathematics and beyond.

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ENHANCING MATHEMATICS EDUCATION THROUGH DIGITAL TOOLS AND HANDS-ON ACTIVITIES

Karen Morrison

Maskew Miller Learning

Abstract

This workshop will provide an in-depth look at Maskew Miller Learning digital platform for GET phase mathematics education. Participants will learn how to utilise Mask TV,Maski AI Tutor, and Maski Videos to enhance their teaching practices and support learner engagement. The session will include hands-on activities to ensure participants can effectively integrate these tools into their classrooms.

Motivation

This workshop aims to introduce teachers to Maskew Miller Learning's digital platform designed to support both teachers and learners in the GET phase. The platform includes innovative features such as Mask TV, Maski AI Tutor, and Maski Videos on concept explainers, which enhance the teaching and learning experience.

Description of Content

• Introduction to Maski Maths Digital Platform: Overview of the platform and its features.
• Mask TV: Interactive sessions and educational broadcasts.
• Maski AI Tutor: Personalised tutoring using artificial intelligence.
• Maski Videos: Concept explainers to aid understanding of complex topics.
• Hands-On Activities: Participants will engage with the digital tools and explore practical applications in their classrooms.

MASKI MATHS CONTENT FOR GET: LEVERAGING DIGITAL TOOLS FOR EFFECTIVE TEACHING AND LEARNING

KAREN MORRISON

Maskew Miller Learning

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PROBLEM SOLVING IN THE FURTHER EDUCATION AND TRAINING

Susan Carletti and Jenny Campbell

The Answer Educational Publishers

Abstract

Are you an FET Maths educator? Are you wanting to help your learners develop their problem-solving skills, and become more confident at answering Level 4 questions? Are you looking for questions that you can use in your classroom that will extend your learners? If yes, then this workshop is for you. We will work through some Level 4 questions. In FET, Level 4 questions count 15%, so learners need to be exposed to techniques they can use. They also need to be encouraged to try problem solving questions, instead of shying away from them. The best way to gain confidence in problem solving is to do it in a non-threatening environment where you have time to play with the questions.

Motivation for the workshop

I am passionate about problem solving and really enjoy helping others to see the fun in it. The joy you see on a learner's or an educator's face as they see the beauty in a particular problem makes teaching so satisfying. In this workshop we will look at Level 4 questions in Grade 10, 11 and 12. These questions cover a variety of topics and are all suitable for use when a particular section is being taught.

Target audience: Grade 10, 11 and 12 educators (FET)
Workshop Duration: 2 hours
No. of participants: There is no maximum number.

Description of content of workshop

We will work through various problems. We will start with Grade 10 questions covering Algebra, Patterns, Functions, Trigonometry, Analytical Geometry, and Euclidean Geometry. We will then work through questions from the same content with Grade 11 work, and then the same for Grade 12, where we will add a Calculus question. Problem solving skills are essential to start learning as soon as possible, as they will help not only with Mathematics, but also any discipline that requires logical thought. The workshop will be hands-on, with educators working through the questions. The worksheet that will be used has been attached.

FOSTERING MENTAL MATHEMATICS COMPETENCE IN GRADES TWO AND THREE

Sehowa Kate, Shemunyenge Taleiko Hamukwaya, Anthony A Essien & Sameera Hansa

University of the Witwatersrand

ABSTRACT

In South Africa, as in many other countries around the world, learner performance in mathematics remains low and is yet to reach the "expected" level. This low performance is often compounded and attributed to the inefficient strategies (seen through classroom practices) used in teaching and learning which hinders learners' progression to efficient calculation strategies and in turn hinders their ability to solve problems that involve higher number ranges. One of the inefficient strategies is persistent counting in ones in the foundation phase which leads to challenges in grasping the base-ten numeration system and the place value understanding. Grounded in Morrison's (2018) doctoral research, the BTT framework integrates constructs of Freudenthal's theory of number structuring and mental mathematics strategies such as the jump and bridging through ten strategies. These approaches aim to develop learners' number sense and foster efficient mental calculation. The purpose of the research and development project called Base Ten Thinking and Mental Starters Assessment Project at Wits University is to investigate the use of efficient strategies in teaching the bridging through ten and jump strategies in Grades 2 and 3 as opposed to finger counting and use of tally marks which often leads to errors with an increasing number range. The overall purpose of this workshop is to develop teachers' competence in teaching mental mathematics using the bridging through ten and jump strategies in Grades 2 and 3.

MOTIVATION OF THE WORKSHOP

A number of teachers in the foundation phase are faced with the challenge of enhancing learners' counting skills, together with getting them to understand the concept of place value, especially in the foundation phase. This workshop is part of the teacher training(s) provided by the Wits Maths Connect Primary Project, under the SA Numeracy Chair. The project carries out training in one of the provinces in SA in which there is a dominance of one indigenous language. AMESA is a good platform for us to reach out to a number of teachers who need support in (learners') developing competence in mental mathematics. The main driving force for this workshop is that there is a significant drop in learners' performance in the foundation phase, in mathematics due to the ineffective strategies counting strategies used. Additionally, a platform like AMESA presents an opportunity to expand our work beyond the one province we work in.

TARGET AUDIENCE: Foundation Phase teachers, but especially Grades 2 and 3 Teachers
MAXIMUM NUMBER OF PARTICIPANTS: 80. Seated in groups of 3-5
DURATION**:** 2 hours
WHAT WILL BE DONE DURING THE WORKSHOP: During the workshop, we will focus on two key strategies as seen in the workshop materials below.

BASE TEN THINKING

SESSION 1: BRIDGING THROUGH TEN (BTT)

SESSION OBJECTIVES:

Fluency: Answer btt calculations instantly.

Strategic calculating: Use efficient btt strategies (using the friendly number, 10) to answer problems.

Strategic thinking: Use relationships to answer btt questions.

SESSION OVERVIEW:

In this session, we will look at different ways of developing fluency in strategic calculating and thinking to solve btt problems. We will do this by looking at different questions and also discuss possible questions that participants can come up with.

TASK:

Problem: $36 + 7 =$

What warm-up exercises linked to the task would you use to prepare learners to solve the question

How would you use the number line to enhance learners' understanding of the ways to solve this question?

What is the specific correct language demands of the task within the maths register?

GROUP DISCUSSIONS**:** Each group discusses ways of teaching the btt question to Grades 2 and 3 learners.

WHOLE CLASS DISCUSSION**:** The facilitator will lead a whole class discussion based on the group discussion.

SESSION 2: JUMP STRATEGIES (js)

SESSION OBJECTIVES:

Fluency: Answer js calculations instantly.

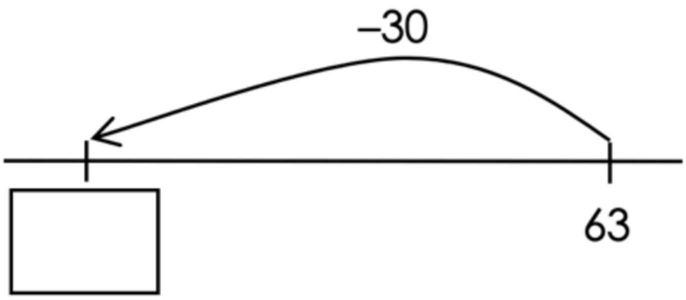
Strategic calculating: Use efficient js strategies (using the friendly number, 10) to answer problems

Strategic thinking: Use relationships to answer questions.

SESSION OVERVIEW:

In this session, we will look at different ways of developing fluency in strategic calculating and thinking to solve js problems. We will do this by looking at different questions and also discuss possible questions that participants can come up with.

TASK:
Problem:



How would you teach this problem to Grades 2 and 3 learners?
How else would you represent the question?
What is the specific correct language demands of the task within the maths register required?

GROUP DISCUSSIONS*:* Each group discusses ways of teaching the js question to Grades 2 and 3 learners.

WHOLE CLASS DISCUSSION: The facilitator will lead a whole class discussion based on the group discussion.

PROMOTING RELEVANCE AND ENGAGEMENT IN GRADE 12 MATHEMATICS CLASSROOMS

Sifundo Moloi and Nondumiso Mnyamana

Marang Education Trust

Abstract

Teachers often struggle to tailor their teaching to suit the diverse needs of learners. The struggle is with accommodating learners with difficulties as well as those who are fast and need little or no support from the teacher. Teachers usually teach in the 'middle', and this leads to systemic barriers for some learners, who are not afforded opportunities to achieve according to their potential. The education system, especially in the FET phase because of the fixation with the grade 12 NSC results, is often designed to be teacher-centred with emphasis placed on teachers completing the ATP on time, often at the expense of learners. As a result, curriculum differentiation is neglected in lessons and the focus shifts to the percentage of learners who pass to the next grade and learners who are successful in the NSC examinations, rather than addressing the diverse needs and abilities of learners. Pre-service training tends to focus on ensuring that Mathematics teachers qualify with solid content and pedagogy, with practical inclusive teaching, learning and assessment strategies often underplayed. This often impacts on the teachers' ability to ensure equitable mathematics instruction in a classroom setting. This workshop will assist educators with inclusive teaching, learning and assessment strategies so as to cater for diversity in the Grade 12 Mathematics classroom.

Description of content of workshop

In the workshop, teachers will be introduced to an inclusive approach that foregrounds curriculum differentiation. The workshop will explore teaching methodologies, assessment strategies and tracking, learner and parental engagement. Each theme will include a corresponding group activity to ensure an interactive workshop experience. Additionally, a short demo lesson will showcase how to implement curriculum differentiation in a grade 12 classroom setting, using probability as an example.

Time frame

Content	Time allocation
A brief introduction of the workshop facilitators. Outlining the purpose of the workshop.	5 min

A Brief background of the importance of implementing curriculum differentiation in a lesson. Short demo lesson on the implementation of curriculum differentiation using the fundamental counting principle as an example.	15 min
Formal and Informal Assessment – A short presentation on how to differentiate assessment by considering all cognitive levels as outlined in CAPS. Group activity on the setting of levels 1 – 4 questions on the fundamental counting principle on codes and passwords. Feedback in plenary.	15 min
Short role play on how to engage with a learner based on the colour coded tracking marksheet on their term progress. This will be consolidated with an overview of how to track learner performance, how to engage both learners and parents on learner performance and the importance of introducing a reward system to motivate learners to stay focused and push to achieve to their potential.	15 min
Questions, answers, reflections and discussions	10 min

WORKSHOP ACTIVITIES

GRADE 12 FUNDAMENTAL COUNTING PRINCIPLE

DEFINITION

The fundamental counting principle is a principle used to determine the number of different ways to accomplish different tasks. It states that:

If there are m ways to perform a task and n ways to perform a second task, the total number of different ways in which both tasks can be performed is $m \times n$.

This can be extended to more than two tasks.

Fundamental Counting Principle (Codes)

Short Demo Lesson

Activity 1

In groups discuss the following and prepare to share in the plenary. (5 min)

1. Where are number passwords used in our daily life? Name as many places as you can.
2. Use the numbers (0-3) in front of you to create as many two-digit codes as you possible can. The *repetition* of digits in the code *is not allowed*.
3. Use the numbers (0-3) in front of you to create as many two-digit codes as you possible can. The *repetition* of digits in the code is *allowed*.

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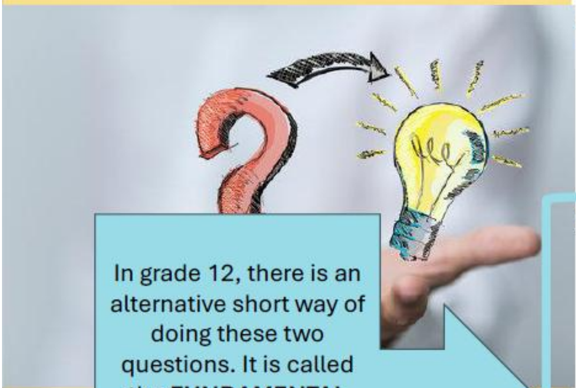

In grade 12, there is an alternative short way of doing these two questions. It is called the **FUNDAMENTAL COUNTING PRINCIPLE.** (m x n)/ (choices x choices)

Activity 1: Answers

1. Cellphone passwords, apps, bank pins, combination padlocks, alarm systems etc.
2. 01,02,03,10,12, 13,20,21,23,30, 31,32 **= 12 codes**
3. 00, 11, 22, 33, 01, 02, 03, 10, 12, 13, 20, 21, 23, 30, 31, 32, **= 16 codes**

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Quick Steps

1. Always *READ* the statement before you answer any question.
2. Underline *KEY WORDS* in the statement using colored pens or highlighters. Key words will include things such as the numbers to be used in the code (people) , the number of spaces (chairs) in the code, whether repetition is allowed or not allowed and some restrictions to the first and last spaces in more complex questions.
3. Remember: We are not interested in the actual individual codes when we do calculations. We are only interested in the total number of codes.

2. 0 ; 1 ; 2 ; 3 repetition not allowed

$$4 \times 3 = 12$$

3. 0 ; 1 ; 2 ; 3 repetition is allowed

$$4 \times 4 = 16$$

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Activity 2

In groups discuss and formulate questions on the fundamental counting principle in codes. Prepare to present in the plenary. (10 min)

1. Formulate four level 1 questions. (Groups 1)
2. Formulate three level 2 questions. (Groups 2 and 3)
3. Formulate two level 3 questions. (Groups 4 and 5)
4. Formulate two level 4 questions. (Group 6)

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Activity 2: Answers

1. Level 1 Questions Examples

1.1 Given the numbers: 0,1,2,3,4,5

a) How many three-digit codes can be formed if repetition is allowed ? (1)

b) How many three-digit codes can be formed if repetition is not allowed? (1)

1.2 Given the numbers 0 – 9, how many four-digit codes can be formed:

a) if numbers may be repeated ? (1)

b) if numbers may not be repeated? (1)

2. Level 2 Questions Examples

2.1 Given the numbers: 0 to 7

a) How many three-digit numbers can be formed if the first digit may not be 2 and the numbers may not be repeated? (2)

b) How many three-digit numbers can be formed if the second digit must be 4 and the numbers may be repeat? (2)

c) How many three-digit numbers can be formed if the numbers must be even and repetition is allowed.

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3. Level 3 Questions Examples

3.1 A five-digit code is made up by using four randomly selected digits from 0 to 8. How many codes can be formed if the code must be an even number that is greater than 70000. Digits may repeated. (3)

3.2 Kwa-Zulu Natal has a new number plate system that uses two letters, two digits and two letters. The following is an example of the number plate:

GH – 58 - ZN

How many number plates can be made using digits from 0 – 9 and using letters excluding vowels if letters can be repeated and digits can not be repeated? (3)

4. Level 4 Questions Examples

4.1 A three-digit number is made up by using three randomly selected digits from 0,1,2,3 4,5,6,7,8 and 9. No digit may be repeated.

4.1.1 Determine the total number of possible three-digit numbers, both even and greater than 600, that can be formed. (4)

4.2 A four-digit code is required to open a combination lock. The code must be even-numbered and may not contain the digits 0 or 1. Digits may not be repeated.

4.2.1 Calculate the probability that you will open the lock at first attempt if it is given that the code is greater than 5000 and the third digit is 2. (5)

Topic tests and tracking of learner performance

- Have topic tests, and a pre-quarterly tests in addition to formal tests.
- Make sure to cover concepts that may have been taught in previous grades, but still need to be assessed in the current grade.
- Learners' performance tracked using different colour codes for each test result:

1. **Red** = failed, below 30% for test
2. **Orange** = Worrisome pass (30% - 39%)
3. White = Average (40% - 69%)
4. **Green** = Highflying pass (70% - 100%)

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EXTRACT FROM GRADE 12 TERM 2 MARKSHEET

Colour-coded marksheet

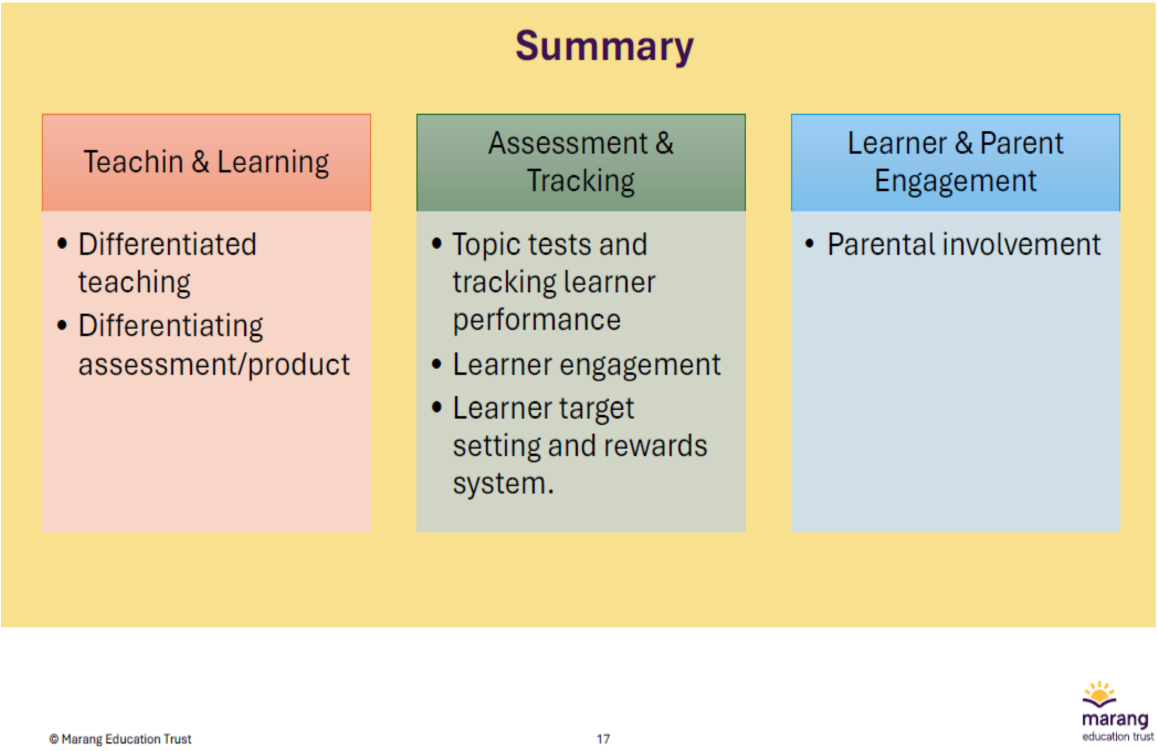
NO.	SURNAME	NAMES	2023 TERM 1	EU GEO GR11 (50) IF 14/04	EU GEO GR 12 (50) IF 21/04	CALCUL (50) IF 05/05	EUCLID + CALC (50) F 12/05	ANALY GEO (50) IF 23/05	P1 + P2 (150) & (150) 09/05 &12/05	TOTAL JUNE COM	GROUP	TERM 2 TARGET
1	QQQQQQ	AA	47%	17	16	07	20	15	43+15	58	G2-1	55%
2	GGGGGG	BB	48%	35	30	24	38	28	84+91	175	G2	60%
3	CCCCCC	CC	44%	30	27	16	13	27	70+53	123	G2	50%
4	EEEEEE	DD	48%	43	31	40	40	41	92+81	173	G2	65%
5	PPPPPP	EE	21%	08	13	06	28	13	21+23	44	G1	30%
6	SSSSSS	FF	24%	16	09	06	10	16	24+25	49	G1	30%
7	ZZZZZZ	GG	18%	12	08	05	05	05	20+19	39	G1	30%
8	VVVVVV	FF	43%	26	24	11	33	23	50+55	105	G2-1	50%
9	NNNNNN	HH	39%	35	32	25	40	25	95+64	159	G2	60%
10	JJJJJJ	II	39%	24	20	18	23	15	90+36	126	G2	50%
11	PPPPPP	JJ	05%	02	07	00	-	00	14+09	23	G1	30%
12	KKKKKK	KK	51%	37	21	23	10	35	77+57	134	G2	60%
13	TTTTTT	LL	11%	22	18	16	03	14	43+31	74	G1	30%
14	AAAAAA	MM	17%	17	13	11	18	14	26+19	45	G1	30%
15	BBBBBB	NN	35%	-	13	18	25	23	64+45	109	G1	45%
16	YYYYYY	OO	68%	36	28	37	25	37	90+79	169	G3	65-70%
17	WWWWWW	PP	45%	35	33	17	33	34	79+46	125	G2	50%
18	CCCCCC	QQ	44%	36	31	21	28	23	62+39	99	G2-1	60%
19	LLLLLL	RR	25%	41	28	20	38	24	42+50	92	G1	60%
20	FFFFFF	SS	47%	43	18	31	28	46	87+65	152	G2	60-70%
21	BMBMBM	TT	22%	06	16	15	23	14	33+22	55	G1	30%
22	LPLPLP	UU	61%	37	26	29	25	43	91+60	151	G3	65-70%
23	YOYOYO	VV	71%	42	44	35	40	50	117+96	213	G3	70-80%

Learner target setting and reward system

- •Request learners to set a target at the beginning of each term
- •Reward learners who reach their target.
- •Learners can be given different tokens depending on their performance.
- •Categories of tokens: top achievers, learners who have reached target, most improved etc.
- •Ensure that as many learners as possible, especially those in the struggling group are kept motivated during the year.

15

marang education trust



Nature of the workshop

Each learner is unique in terms of their interests, learning styles, the pace at which they learn, and cognitive capacities. Hence, to provide equitable mathematics education and address barriers to learning, teachers should be aware of their learners' learning styles and adapt their teaching approaches to suit each learner's needs. This session will therefore provide educators with the strategies they need to implement inclusive teaching practices that enable every learner in the classroom to realise their potential. The aim is to share insights and enhance teachers' abilities to support learners who can function at a concrete level without excluding those who can function at an abstract level or vice versa.

Content

In the session, the following will be demonstrated using the activities above:

- How teachers can develop inclusive classrooms by teaching the same content to all the learners but provide alternative routes to master it.
- How to facilitate opportunities for independent learners to read to learn, to gather information on their own and present and share their insights.
- How formal and informal assessment can be differentiated to cater for the diverse needs of learners and ensure equitable mathematics instruction.
- How to track learner performance, by involving both the learner and parent in the process.

- Using a reward system to motivate learners to set personal academic goals and achieve according to their abilities and set targets.

Conclusion

Diversity should not be viewed as a barrier, but rather as an opportunity to maximise each learner's potential. Rather than being perceived as a hindrance, diversity ought to be welcomed and leveraged to fully realise the potential of every learner. We will be helping teachers ensure equitable and inclusive classrooms by holding this session.

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PROPERTIES OF 2-DIMENSIONAL SHAPES AT INTERMEDIATE PHASE

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Kgwadu Primary School, Capricorn North District

Abstract

This tangram-based workshop is an effective way to address misconceptions about the properties of 2D shapes. By actively manipulating the seven pieces of a Tangram, learners gain a hands-on understanding of key geometric concepts such as symmetry, congruence, and the relationships between shapes. The tactile nature of Tangrams helps make abstract concepts more concrete, allowing learners to visually explore properties like angles, area, and perimeter. They can explore how triangles can combine to form squares or how the area of a large shape can be broken into smaller, congruent pieces. This helps in understanding key properties like congruence, symmetry, and the relationship between shapes. They can physically see how triangles form other shapes, helping them link geometric vocabulary to their visual and tactile experiences.

Tangram exercises encourage critical thinking and problem-solving, as learners rearrange pieces to form specific shapes, which helps them correct misconceptions in real-time. They also provide a unique opportunity to explore transformations like rotation, reflection, and translation, reinforcing these ideas through practical application. Additionally, the workshop promotes the use of geometric vocabulary, such as parallel, perpendicular, and congruent, which strengthens students' understanding of terms in context.
By working in groups, learners can collaborate, discuss, and learn from each other, fostering a supportive learning environment. The interactive nature of Tangram activities keeps learners engaged and motivated while reinforcing important geometry concepts. Overall, this engaging workshop and effective tool for building a deeper understanding of 2D shapes and addressing common geometric misconceptions.

Description of content of workshop

Teachers will explore the properties of 2D shapes through seven Tangram pieces and how they can be used to teach key concepts such as **symmetry**, **congruence**, **angles**, and **transformations**.

There will be **hands-on Activities**:

- **Shape Creation**: Teachers will use Tangram pieces to form various shapes and figures.
- **Symmetry and Transformations**: Activities will involve exploring symmetry, reflection, and rotations.

Discussion and Reflection: After practical activities, teachers will discuss their experiences, ask questions, and share ideas for implementing Tangram-based activities in their classrooms. By the end of the session, teachers will have a deeper understanding of geometric concepts and hands-on techniques they can use in their classrooms to engage students in learning geometry.

♣ **What will be done in the workshop? How will the time slot be broken up?**

Teachers to engage in hands-on activities aimed at enhancing their understanding of **2D geometric concepts** and equipping them with practical strategies to implement in their classrooms. They will actively participate in creating shapes and exploring geometric transformations.

The workshop is structured for an **hour**, with **75% of the time (45 min)** allocated to presentation and practical activities. During this time, teachers will engage in various exercises that demonstrate how Tangrams can be used to teach geometry effectively. The remaining **25% (15 min)** will be dedicated to discussion and questions, allowing teachers to reflect on their learning, ask questions, and share insights for implementation in their respective schools.

♣ **The activities and worksheets to be used in the workshop (maximum 8 pages)**

Tangram Shape Creation

Objective: Have teachers and students use the seven Tangram pieces to form specific shapes or figures.

Activity: Give teachers and students a set of Tangram pieces and challenge them to create a specific shape, such as a square, triangle, or more complex designs like animals or objects (e.g., a duck, house, or boat).

Purpose: This activity promotes spatial reasoning, symmetry, and the understanding of how shapes fit together.

Shape Decomposition and Composition

Objective: Help teachers and students explore how larger shapes are made up of smaller ones.

Activity: Ask teachers and students to take a large shape (such as a square or rectangle) and break it into smaller Tangram pieces. Alternatively, have them combine the seven pieces to form new shapes.

Purpose: This activity teaches teachers and students about **decomposition** (breaking a shape into smaller parts) and **composition** (combining smaller shapes to form larger ones), which are key concepts in geometry.

Symmetry Exploration

Objective: Explore the concept of symmetry using the Tangram pieces.

Activity: Challenge teachers and students to create symmetric designs by reflecting or rotating the pieces. teachers and students can also try to identify shapes within the Tangram set that have lines of symmetry.

Purpose: This activity reinforces the concept of symmetry and reflection and helps students develop an intuitive understanding of geometric transformations

♣ **An abstract describing the level, nature and content of the workshop (200 words**).

The workshop is designed for intermediate-level learners in grade 4-6, with the goal of deepening their understanding of 2D geometric concepts. This hands-on workshop focuses on key topics such as symmetry, congruence, geometric transformations, and the relationships between different shapes. Learners will use Tangram pieces, seven geometric shapes that can be rearranged to form various figures allowing for an interactive learning experience.

The nature of the workshop is learner-centred and engages students in a variety of activities. This is guided by the mathematics teaching and learning framework for South Africa, where a learning centred mathematics classroom is characterised by a culture of interaction between teachers and learners in the process of doing These include creating specific shapes, decomposing complex figures, and exploring transformations like rotation, reflection, and translation. Learners will also examine how different shapes can be combined to form new ones, helping to solidify their understanding of composition and decomposition.

The content covered includes properties of 2D shapes in particular: symmetry exploration, the calculation of area and perimeter. Through these activities, students will not only strengthen their spatial reasoning and critical thinking skills but also address common misconceptions about geometric properties and relationships.

Ultimately, the workshop will offer an engaging, interactive environment that encourages creativity while providing students with a deeper, more intuitive understanding of geometry. The workshop is not only a valuable resource for learners but also an excellent professional development opportunity for teachers. It helps them refine their teaching techniques, enhance classroom engagement, better understand learner misconceptions, and improve their own grasp of geometric concepts. By incorporating Tangram activities into their teaching practice, teachers can foster a more interactive, learner-centred, and creative learning environment.

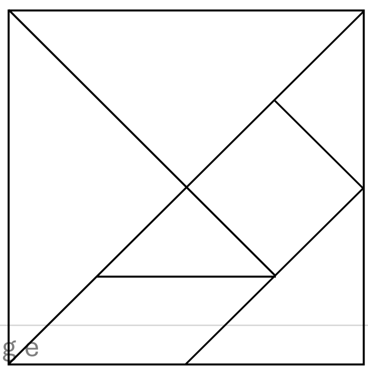
ACTIVITIES FOR THE WORKSHOP

SECTION A

The first activity will begin with a brief introduction to the Van Hiele model of geometric thinking, which will serve as a foundation for the learners understanding.

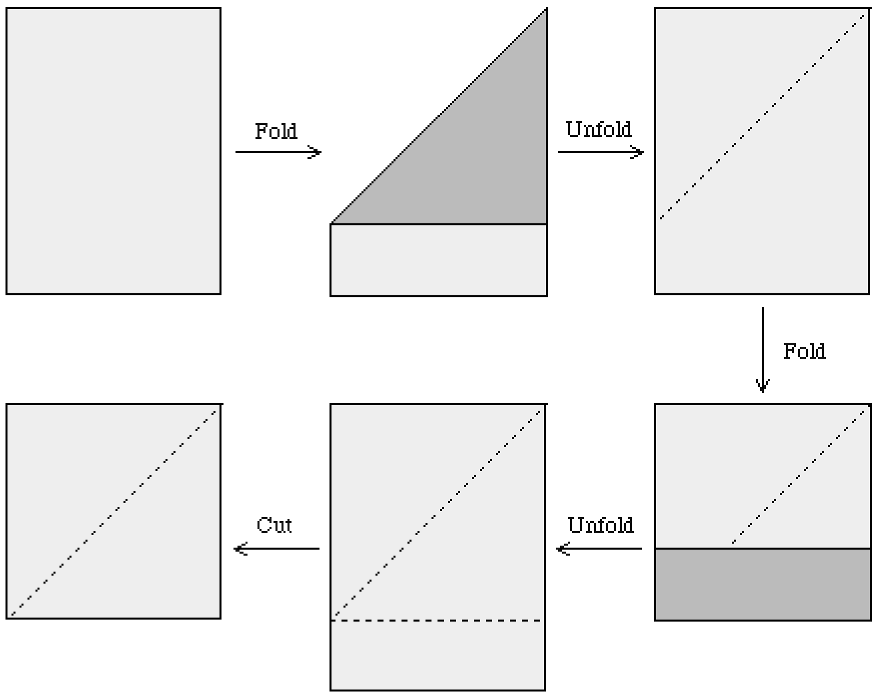
SECTION B
MAKING A TANGRAM

Identify and name each of the seven pieces in the tangram.

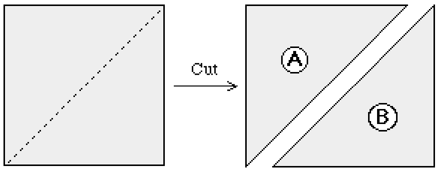


2. You can make your own set of tangrams from a single piece of paper. Just follow these simple steps:

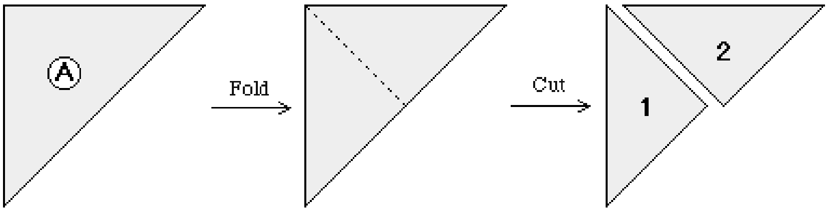
A. Fold a rectangular piece of paper so that a square is formed. Cut off the extra flap.



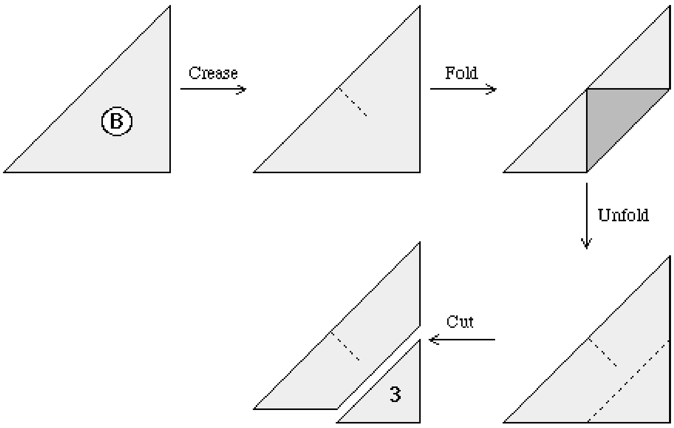
B. Cut the square into two triangles.



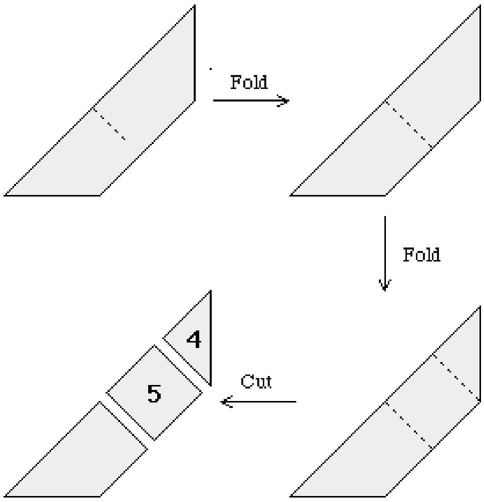
C. Take one triangle and fold it in half. Cut the triangle along the fold into two smaller triangles.



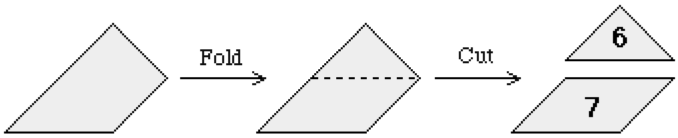
D. Take the other triangle and crease it in the middle. Fold the corner of the triangle opposite the crease and cut.



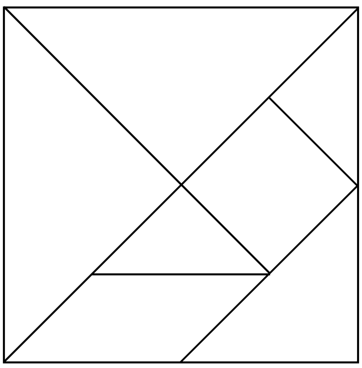
E. Fold the trapezium in half and fold again. Cut along both folds.



Fold the remaining part and cut it in two.



Now arrange your 7 pieces to form a tangram.



SECTION C

1. <u>COMPOSING AND DECOMPOSING OF SHAPES</u>

a) Use figures 3, 5 and 7 to make a square. How do you know that the figure you have made is a square?

b) Use figures 3, 5 and 6 to make a rectangle. Is there more than one way? How do you know the new figure is a rectangle?

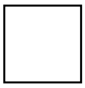
c) Use pieces 1, 3, 4 and 5 to make a rhombus.

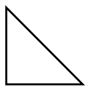
d) Use pieces 1 and 7 to make a trapezium.

e) Use figures 3, 4 and 5 to make a triangle the same size as figure 2. Is there only one way?

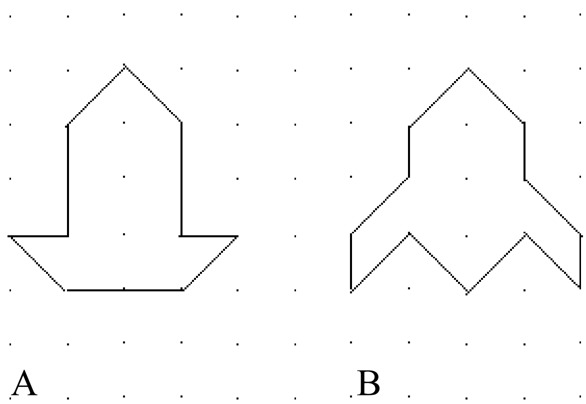
f) Yusuf says it is not possible to make the square made with figures 1 and 2 **without using figure 1 or 2**. Do you agree with Yusuf?

2. CALCULATING AREA USING TANGRAMS

If the area of is 1  square centimeter (1 cm^2)

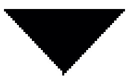
a) What is the area of ? 

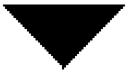
b) Haadiya says she can find the area of each of the following drawings by using the area of both the square and triangle showed above. Can you help her?



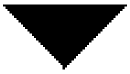
Area of A =
Area of B =

Symmetry and congruency

1. How many  are in  ?

2. How many  are in  ?

3. How many  are in  ?

4. If  = 1, then  =

5 . If  = 1, then  =

Two Hour Workshop

APPLICATIONS OF MATHEMATICAL MODELLING AS AN INSTRUCTIONAL STRATEGY IN SOUTH AFRICAN SCHOOLS

Duncan Mhakure[1], Wisani Hlangwani[1] & Miranda Moodley[1]

[1]University of Cape Town

Abstract

A growing number of countries, including South Africa, have integrated mathematical modelling (MM) into their national curriculum (Huang et al., 2021; Jakobsen & Mhakure, 2024). In the context of this workshop, MM is defined as "the process leading from a[n] [authentic] problem situation to a mathematical model" (Blum, 2002, p. 153). A mathematical model can be defined as: a formula; an equation; or a function – representing the relationship between variables. In South Africa, while MM is an important focal point in the National Curriculum and Assessment Policy Statement (CAPS) document, its applications in the teaching of mathematics is limited. In this workshop, we argue that mathematics teachers can use MM as teaching tool to: structure the learning process and help learners to develop new mathematical modelling competencies; and introduce and develop mathematical concepts through solving tasks as an alternative to the traditional short question and answer model, which is preferred in the current high-stakes examination environment (Mhakure & Jakobsen, 2021; Yvain-Prébiski & Chesnais, 2019). Therefore, the aims of this workshop are to (a) review the status of MM as an instructional strategy to mathematics in schools; (b) illustrate through examples how MM can be applied in the teaching of mathematics; (c) illustrate how model-eliciting examples can be designed from textbooks and be solved; and (d) identify the challenges associated with the use of MM as a teaching tool within the South African mathematics teaching context

Target audience:	Mathematics teachers interested in mathematical modelling.
Duration:	2 hours
Maximum number:	60
Participants:	50 participants
Motivation for the workshop:	A growing number of countries, including South Africa, are integrating mathematical modelling in the teaching and learning of mathematics in schools

Workshop plan

1. Introduction to mathematical modelling framework presentation - 10 mins
2. Mathematical modelling tasks – 20 mins
3. Solving mathematical modelling tasks with teachers – 80 min
4. General discussion – 10 mins

. **Mathematical Modelling Tasks**

The following MM tasks are solved by teachers during the workshop.

Task 1: Theoretical task – hay bales

The diagram above shows a pile of bales of hay in a field. A farmer approaches you and asks you to calculate the height of the pile of bales.

Task 2: Theoretical task - *Juiciest Beef Burger Patty*

Tired of dry, lacklustre burgers that taste like cardboard? Try my Gogo's (grandmother) secret ingredients recipe – posits Thabo. The following recipe will sort you out when it comes to juicy and tasty burger patties (meaty part of a hamburger). These are juicy, and spices can easily be added or changed to suit anyone's tastes. Baste frequently with your favourite barbeque sauce. If you find the meat mixture too mushy, just add more breadcrumbs until it forms patties that hold their shape.

Gogo's secret ingredients (4 – 6 servings)

- $1\frac{1}{2}$ kilograms beef
- 1 egg
- 3/4 cup dry breadcrumbs
- 3 tablespoons milk
- 2 tablespoons Worcestershire sauce
- 1/8 teaspoon cayenne pepper
- 2 buds of garlic

Thabo Mali is planning to host a group of friends to watch a soccer derby match on Saturday. He plans to serve them burgers during half time in the game. Thabo knows his Gogo's recipe for making very juicy patties only caters for 4 to 6 people, and he suspects that he will have between 6 to 9 friends visiting on Saturday. He would like advice on how he should adapt the burger patty ingredients above to cater for his friends, come Saturday.

Task 3: The Historic Hotel Problem

Mr. Sipho Mthembu has just inherited a historic hotel located in the KwaZulu-Natal Midlands. He would like to continue running the hotel, but he has limited experience in hotel management. The hotel has 80 rooms, and Mr. Mthembu was told by the previous owner that all 80 rooms are usually booked when the daily room rate is R1 200. He also learned that for every R20 increase in the room price, one fewer room is booked. For example, if he charges R1 220 per room, only 79 rooms are booked. If he charges R1 240, only 78 rooms are booked, and so on. Each room that is booked costs R80 per day for cleaning and maintenance.

1. Find a mathematical model that can be used to charge each room to make the most profit?
2. What the best room price in the future if the room prices or maintenance costs change?

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SUPPORTING TEACHERS TO IMPLEMENT MOTHER TONGUE-BASED BILINGUAL EDUCATION IN MULTILINGUAL MATHEMATICS CLASSROOMS

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University of the Witwatersrand, Johannesburg

Abstract

Closing language-induced achievement gaps in the early grades is of utmost importance in the teaching and learning of concepts with understanding. In bi/multilingual contexts where learners come to class with varying proficiencies in their home languages as well as in the LoLTA, language-induced achievement gaps are even more prominent necessitating a more urgent need to attend to language practices that can enhance mathematical learning and thus contribute towards closing such gaps. For this reason, the overall purpose of the research and development project called Language and Mathematics in 'Early' Grade (LMEG) at Wits University is to investigate language practices in early grade mathematics in order to inform 'best practice', specifically translanguaging practices in the early grades in the context of the newly promulgated Mother tongue-based bilingual education (MTbBE). The introduction of MTbBE ensures that there is no abrupt switch from mother tongue to English, but that this transition is done progressively. The overall purpose of this workshop with intermediate phase teachers, particularly Grade 4 teachers, is to develop practices that can enable better epistemic access to mathematics in their classrooms in the context of MTbBE.

Motivation for the workshop

More and more teachers in the intermediate phase face a daunting task of how to manage learners who in the Foundation Phase have been taught through the medium of their home language and who now have to be taught in two languages – the home language and English from Grade 4 onwards. This workshop is part of the training the Wits Maths Connect Project, under the Numeracy Chair carries out in one Province in South Africa. AMESA provides a good forum for us to reach out to many more teachers who are in dire need of being supported to implement mother tongue-based bilingual education (MTbBE). The key driving force for this workshop are; 1) most in not all teacher education programmes in South Africa do not focus on training teachers to teach in more than one language which is now a necessity with MTbBE; 2) Performance in mathematics has been seen to drop in the Intermediate Phase, and the lack of ability to deal with these language issues is one of the contributing factors; 3) The Numeracy Chair's work in this regards has been seen to make impact in one District, and hence AMESA provides a forum for scale up.

Target Audience: Foundation Phase and Intermediate Phase teachers, but especially Grade 4 Teachers
Maximum number of participants: 80. Seated in groups of 7-10 (if possible)
Duration: 2 hours
What will be done during the Workshop: During the workshop, we will focus on two key roles ('jobs') of the teacher in language responsive mathematics teaching as seen in the

workshop materials below.

LANGUAGE RESPONSIVE MATHEMATICS TEACHING

Session 1: Noticing, Supporting, and developing Learners' language needs and their mathematical reasoning

Session objectives:

- How to NOTICE learners' language needs and their mathematical thinking
- How to support the mathematical thinking and the language needs of learners after noticing

Session overview:

In this session, we will look at different ways of NOTICING, or paying attention to learners mathematical reasoning and how learners express this thinking through language. We will do this by looking at different scenarios, and also discussing scenarios that participants can come up with.

Read Excerpt 2 below (Moschkovich, 1999, pp 13-14):

	Teacher: Today we are going to have a very special lesson in which you really gonna have to listen. You're going to put on your· best, best listening ears because I'm only going to speak in English. Nothing else. Only English. Let's see how much we remembered from Monday. Hold up your rectangles .. high as you can [students hold up rectangles] Good, now Who can describe a rectangle? Eric, can you describe it? [a rectangle] Can you tell me about it?
2	Eric: A rectangle has two short sides, and two .. long sides
3	T: Two short sides and two long sides. Can somebody tell me something else about this rectangle? If somebody didn't know what it looked like, what, what how would you say it?
4	Julian: Parallel(o) [holding up a rectangle]
5	T: It's parallel. Very interesting word Parallel, wow! Pretty interesting word, isn't it? Parallel. Can you describe what that is?
6	Julian: Never get together. They never get together [runs his finger over the top length of the rectangle]
7	T: What never gets together?
8	Julian: The parallela the . when they go, they go higher [runs two fingers parallel to each other first along the top and base of the rectangle and then continues along those lines] they never get together
9	Antonio: Yeah!
10	T: Very interesting. The rectangle then has sides that will never meet. Those sides will be parallel Good work. Excellent work. Anybody else have a different idea that they could tell me about rectangles?
11	Student: Another different [unclear] parallelogram
12	T: It's called a parallelogram, can you say that word?
13	Ss: Parallelogram.
14	T: What were you going to say, Betsy?
15	Betsy: Also a parallelogram it calls a rectangle
16	T: A parallelogram is also a rectangle? They can be both?

17	Betsy: Yeah
18	T: Wow, very interesting. Can you convince me that they can be both?
19	Betsy: Because a rectangle has four sides and a parallelogram has four sides
20	T: [unclear]
21	Eric: [unclear] a parallelogram
22	T: You want to borrow one? [a tangram piece] I really want to remind you that you really have to listen while your classmate is talking
23	Eric: Because these sides [runs his fingers along the widths of the rectangle] will never meet even though they get bigger, and these sides [runs his fingers along the lengths of the rectangle] will never meet even though they get bigger And these sides [picks up a square] will never meet [runs his hand along two parallel sides] and these sides will never meet [runs his hand along the other two parallel sides]
24	T: When you say get bigger you mean if we kept going with the line? [gestures to the right with his hand]
25	Eric: Yeah
26	T: Very interesting

Answer the following questions in your groups (each group will summarise their points to the whole class):

1. What practices do you see in this class discussion
2. Name instances where the teacher NOTICED learners' language needs
3. How did the teacher address these language needs (if at all)?

Whole class discussion: The facilitator will lead a whole class discussion based on the group discussion.

Summary

- Professional noticing is a teacher's ability to make *in-the-moment* decisions and perform each of the following three interrelated skills: attending to learners' thinking, interpreting their understandings, and deciding how to respond to both their thinking and understanding (Jacobs et al, 2010).

- The ability to notice the mathematical thinking of learners is an important aspect of the teaching process.
- Teaching learners to communicate mathematically is also an important aspect of the teaching process
- To be able to do all the above, it is important to pay attention to learners' language use during discussions.
- It is also important to engage learners in meaningful mathematical discussions that goes beyond procedures for arriving at the correct answer.

Session 2: Using more than one language to teach/learn mathematics: What does it entail?

Session Objectives:

- To understand the benefits and challenges of using more than one language in the teaching and learning of mathematics in a transition class.
- To understand the importance of supporting learners' participation in classroom discussion
- To understand the importance of not only supporting vocabulary development but ALSO supporting students' participation in mathematical arguments
- To understand the importance of focusing on the mathematical content of the discussion. This is one of the foci points of the next session.

Session Overview:

In this session, we will draw on some of the tasks and excerpts that we have used in previous sections to explore deeper the advantages (and challenges) of using multiple languages in the teaching and learning mathematics in bi/multilingual classrooms.

Task

Consider different mathematical terms in isiZulu.

1. What do they mean literally in English?
2. How do these meanings align to the mathematical meanings (as we know them)?
3. How can we use these meanings to better teach the mathematics concept?
4. Always think of the question: ***What mathematics register related to this task would you want to explore further in your learners' home language?***

Revisit Excerpt 2 and consider **Excerpt 3** below from the same class (Moschkovich, 1999, p.14):

Comparing a rectangle and a triangle: Learners were folding the rectangle and cutting it into a folded triangle and a small rectangle. Holding these two pieces, the teacher asked the learners to tell him how a triangle differs from a rectangle. The discussion below ensured:

56	T: Anybody else can tell me something about a rectangle that is different from a a triangle that's different from a rectangle? Okay. Julian?
57	Julian: I he rectangle has para… parallelogram [running his fingers along the lengths of the rectangle], and the triangle does not have parallelogram
	58 I: He says that this [a triangle] is not a parallelogram How do we know this is not a parallelogram?
59	Julian: Because when this gets .. When they get, when they go straight, they get together [runs his fingers along the two sides of the triangle]
60	So, he's saying that if these two sides were to continue straight out [runs his fingers along the sides of the triangle], they would actually intersect, they would go through each other Very interesting. So, this is not a parallelogram and it is not a rectangle. OK

Answer the following questions in your groups (each group will summarise their points to the whole class):

1. What practices do you see in the above class discussion?
2. Name instances where the teacher NOTICED learners' language needs.
3. How did the teacher address these language needs (if at all)?
4. How does the teacher support discussions in the class?

Whole class discussion: The facilitator will lead a whole class discussion based on the group discussion.

Summary

Instructional strategies that focus more specifically on mathematical discourse:

- Prompting learners for clarification
- Accepting and building on learner responses
- Revoicing learner statements, by interpreting and rephrasing what learners say.
- Using several expressions for the same concept
- Using gestures and objects to clarify meaning.

BUILDING BRIDGES: A GEOMETRIC TRANSFORMATION FROM THE CONCRETE TO THE ABSTRACT

Christiano Mthethwa, Tumelo Mokhele Oratilwe Ledwaba

Olico Maths Education

Abstract

The aim of this workshop is to empower teachers with innovative approaches to teaching geometry that focuses on developing learners' "geometric eye" through incidental and intentional learning experiences. Through a combination of incidental and intentional learning, participants will explore holistic, accessible activities designed to engage the natural curiosity of senior-phase learners. These activities incorporate play, concretisation, conceptual understanding, discovery, and retrieval practice.

Through interactive offline and online activities, and teaching methods, teachers will acquire resources and strategies to enhance their professional skills and students' knowledge of geometry. The workshop includes experimental sessions and team collaboration, enabling teachers to establish inclusive learning spaces that foster student mathematical exploration and creativity.

This workshop enables participants to acquire the necessary skills towards developing dynamic geometry lessons that will actively engage their students. Theory will be connected to practice through this initiative to improve both geometric education and teaching quality for students in their enriched learning environment.

Target Audience: Senior Phase

Duration: 2 Hours

Maximum Number of Participants: 50

Motivation for the workshop

The field of geometry is often perceived, by the learner, as abstract, inaccessible, and disconnected from everyday life. However, geometry is an integral part of our world, from the design of buildings and bridges to the patterns found in nature. This workshop aims to make geometry more accessible, relevant, and engaging for all students, particularly those from diverse backgrounds.

Moreover, this workshop aims to address the lack of geometric understanding among learners by providing them with hands-on experiences to develop their geometric eye. We will focus on creating curated and guided experiences that incorporate incidental and intentional learning, play, concretisation, and discovery. By participating in this workshop, teachers will gain a deeper understanding of geometric concepts and develop the skills to create engaging and interactive learning experiences for their students.

Objectives

1. Develop Geometric Understanding: Equip teachers with a deeper understanding of geometric concepts, terminology, and structures.
2. Make Learning Geometry Accessible: Use play, concretisation, and discovery to make learning geometry more holistic and engaging for teachers and their students.
3. Enhance Teacher Capacity: Provide teachers with opportunities for incidental and intentional learning, enabling them to create similar experiences for their students.

Content Description

1. Workshop

- **Introduction to an Angle as turn(20 mins)**

In this section, teachers will explore how the notion of angles can be concretised by the use of interactive models. Coupled with prompt questions, teachers will also be exposed to ways to optimise incidental learning and ways to use that knowledge to underscore intentional learning and problem-solving.

- **Straight line Geometry (30 mins)**

This section focuses on engaging teachers on the benefits of learning-through-play firstly by using online Kahoot Quiz (https://create.kahoot.it/share/grade-7-geometry-terminology-recap/17308697-93b2-4793-98a3-288019636671) as means of retrieval practice. An offline version of these games will also be demonstrated. Thereafter, teachers will experience how to incorporate learning through discovery by use of interactive Geogebra online tools (https://www.geogebra.org/math/angles#high-school) and will be shown how to achieve the same effects offline.

- **Triangles (20 mins)**

Teachers will engage in a Triangle Snap game that reinforces the classification of triangles by side length, catering to senior phase learners' curiosity. This activity also provides a fun way to practice retrieval.

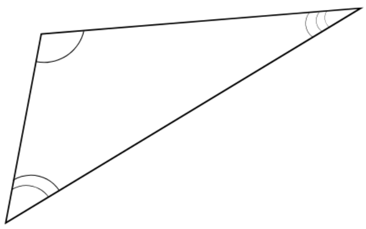
- **Quadrilaterals (30 mins)**

Lastly, by further use of prompt questions and physical models, as well as play with grid-charts, teachers will experience ways in which to provide learners with hands-on experience regarding the properties of quadrilaterals. An online game will be paired with this demonstration to further illustrate ways in which online quizzes can be structured to enhance learners' experiences of learning geometry. (https://kahoot.it/challenge/04121575?challenge-id=d37733c7-6ff6-49a3-9ddd-0fe5fb23037f_1741693767725)

2. Discussion and Q&A (20 mins)

Offline games/activities

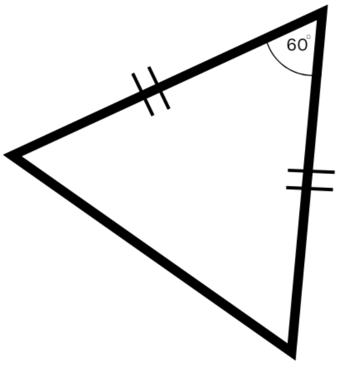
- Triangle snap games



two of the sides are not equal to each other

two of the angles are not equal to each other

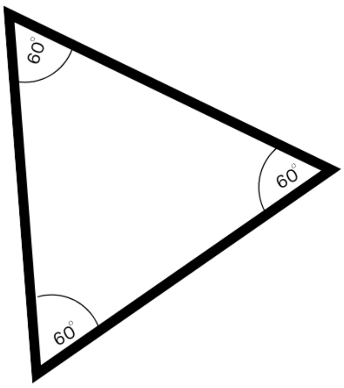
the smallest side is opposite the smallest angle



There are equal angles

There are equal sides

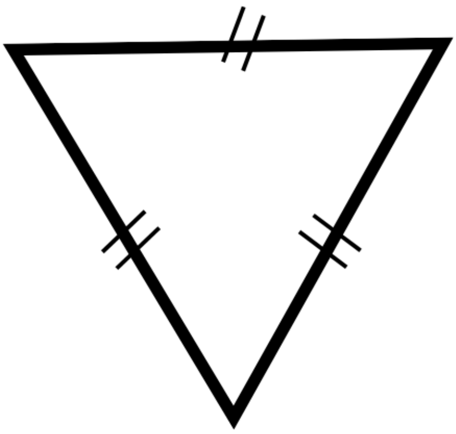
there is more than one line of symmetry



All sides are equal

It is an equiangular triangle

there are only three lines of symmetry

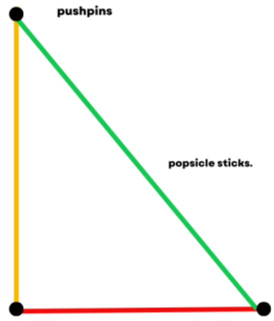


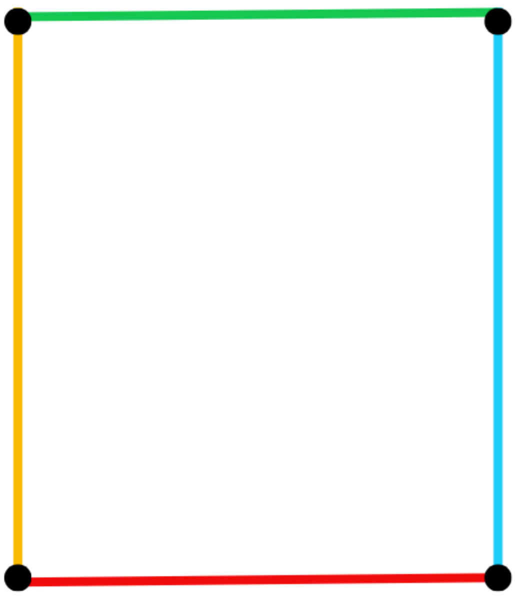
this is called an
equilateral triangle

this shape is regular

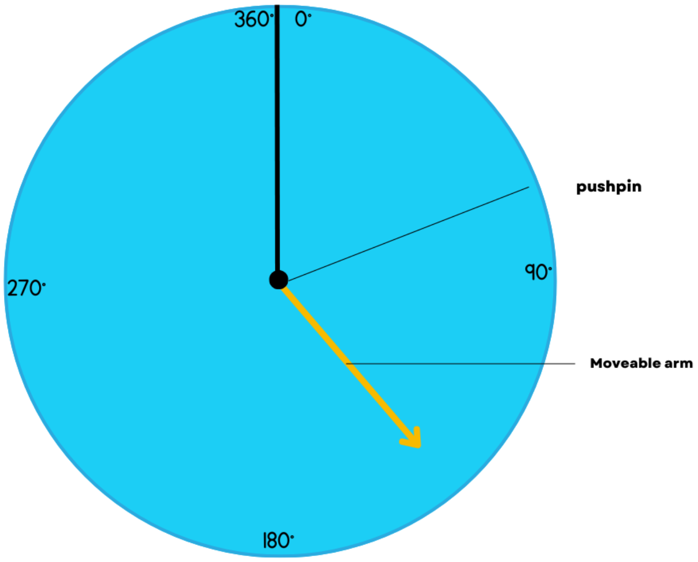
all 3 angles are acute
angles

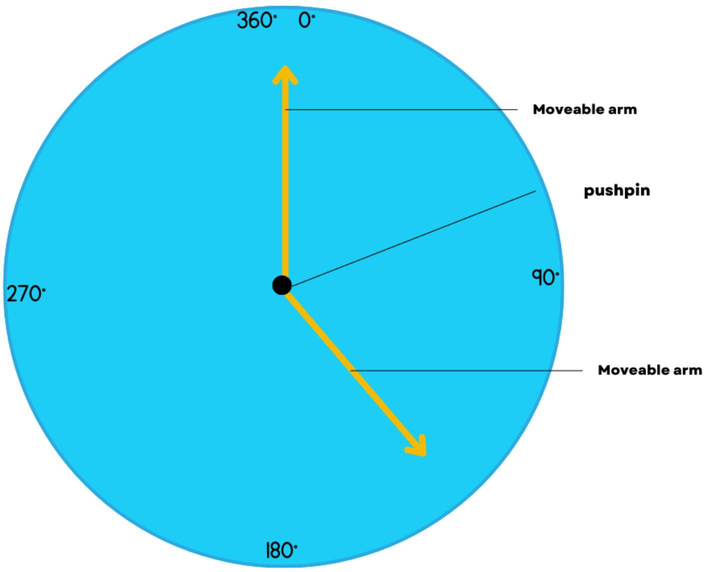
- Interactive physical models to explore angles as turn, properties of quads and properties of quads in relation to triangles





- One model will have all arms the same length
- One model will have two pairs of equal arms
- One model will have one pair of equal arms
- All models should be easy to dismantle and reconstruct to allow learners to create their own quads



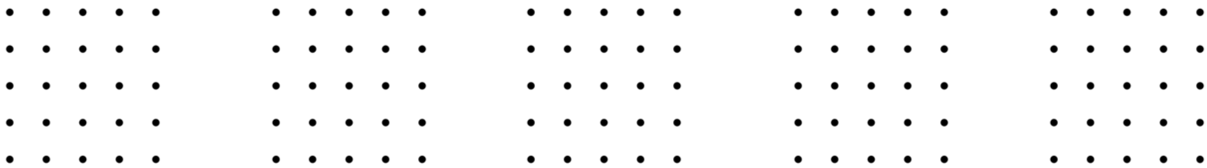


Dotty quadrilaterals 1

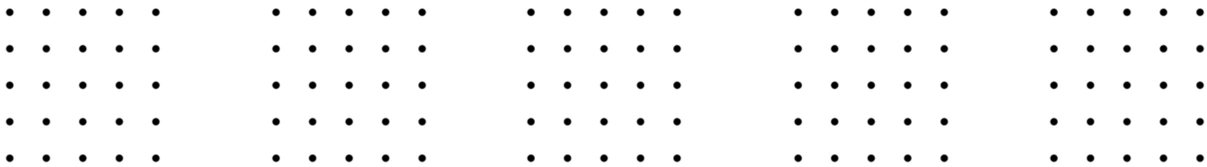
(Acknowledgement: Don Steward donstewardblogspot.com)

join dots to make **quadrilaterals** – one of the vertices *must* be the top left dot (and the other vertices must be on the dots)

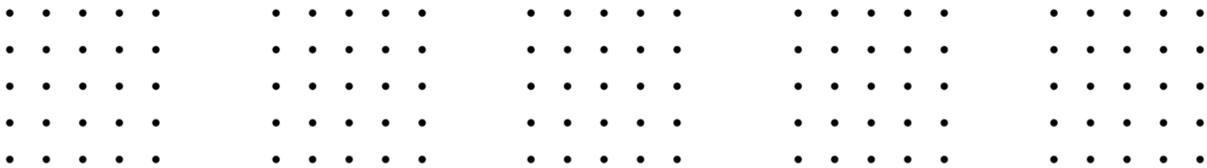
1) biggest square 2) biggest rectangle 3) biggest parallelogram 4) large isosceles trapezium 5) biggest kite



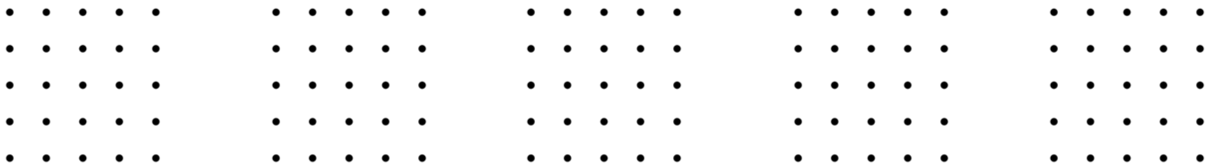
1) smallest square 2) smallest rectangle 3) smallest parallelogram 4) small isosceles trapezium 5) smallest kite



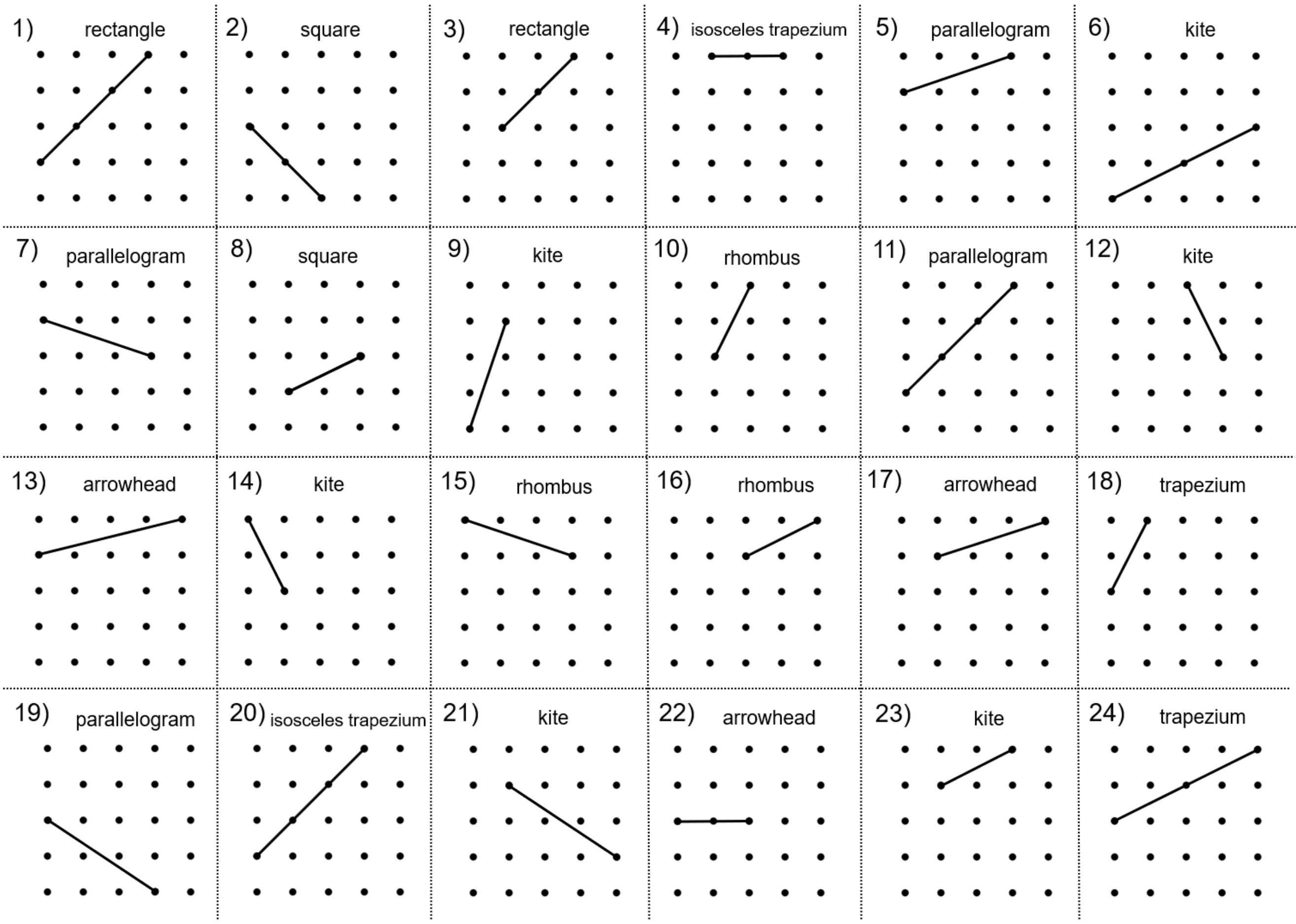
6) biggest rhombus 7) biggest trapezium 8) biggest arrowhead 9) biggest non-special 10) biggest equal diagonals



6) smallest rhombus 7) smallest trapezium 8) smallest arrowhead 9) smallest non-special 10) smallest equal diagonals



join dots to complete these **quadrilaterals** – where there are options, try to find the one on the grid with the largest possible area



Technical Requirements

A fully equipped lab with at least 50 computers, a projector, audio, and an internet connection.

PRACTICAL WAYS TO TEACH SPACE AND SHAPE USING THE BALA WANDE MATERIALS AND METHODOLOGIES

Vuyokazi Mafilika, Tutula Diniso and Ingrid Sapire

Funda Wande

Target audience: Foundation Phase teachers
Duration: 2 hours
Maximum no. of participants: 50
Motivation for the workshop: Why is this workshop important, how will it help participants?

Mathematics education plays a critical role in developing learners' problem-solving skills, critical thinking, and analytical reasoning. However, many learners in South Africa struggle with mathematics, as is evidenced in test results such as on the TIMSS which is an indication of lack of understanding and engagement (Department of Basic Education, 2020; Taylor and Taylor & Taylor, 2017). Research has shown that interactive and inquiry-based learning approaches can improve learner engagement and understanding in mathematics (Hiebert & Grouws, 2007). Teaching in such a manner would support equitable teaching and learning of mathematics in schools. In particular, the development of children's sense of space and shape is important because this develops their cognitive development in relation to spatial awareness which is one of the foundational math skills – developing their visual literacy, mathematical languages and problem-solving abilities.

This workshop aims to address the following questions in a practical way by drawing on the lessons, activities and resources used in the Bala Wande programme:

- *How can I teach concepts related to space and shape in a Foundation Phase classroom in ways that make sense to learners?* and
- *What resources can I use to develop learners' understanding of space and shape?*

The aim is for every teacher who attends the workshop to leave with a better conceptual understanding of space and shape related topics particularly working with 2-dimensional shapes in the context of tessellations, pattern work, creating shapes and reasoning about how shapes can be combined to make other shapes and so on. The workshop will provide teachers with knowledge and practical skills of how to develop Foundation Phase learners' understanding of these topics through the use of sound teaching practices and concrete resources (such as shape cuts outs, tangrams) which can be made easily and for minimal cost by teachers.

The Bala Wande programme is founded on the belief that access to equitable mathematics education is the right of all children in South Africa. Through this workshop, more teachers will have access to equitable teaching and learning opportunities gained from ‘best practice’. All the ‘best practice’ information that will be shared has been gleaned from the research and development done by Bala Wande in South African classrooms. The Bala Wande programme consists of a set of packaged materials which includes a Teacher Guide (TG), which provides day-by-day guidance for the teaching of mathematical concepts, a Learner Activity Book (LAB) with worksheets and games to be used for consolidation of concepts that have been taught in each lesson, and a set of manipulatives for the teacher and learners. Instructional videos also form part of the packaged materials – all of which are aimed at supporting teaching and learning in the classroom. All Bala Wande materials and resources are freely available on the internet for anyone to download and use.

In this workshop we will draw on Bala Wande’s set of open-source materials (freely available on the website https://fundawande.org/) to demonstrate how teachers can use concrete materials to teach space and shape in the Foundation phase. Through modelling and micro-teaching activities – which will involve whole class and group work – we aim to give teachers tangible experience of how to develop learners’ understanding of 2-dimensional shapes. Teachers will be shown how to access Bala Wande materials and will be provided with extra resources and tools to use in their classrooms.

Time	Topic	Activity
15 min	Presenter: Vuyo Introduction to the workshop & presenters Ice breaker	Outline of workshop Handouts
15 min	Presenter: Vuyo and Ingrid Why does learning about space and shape matter for Foundation phase learners?	PPT presentation Turn and talk
25 min	Presenter: Vuyo Geometric pattern work with 2-D shapes • Hands on activities from Bala Wande – pattern work (geometric patterns) • Classroom activities involving 2-D shapes and geometric patterns	BW Video Group activities Feedback
25 min	Presenter: Ingrid Spatial awareness and reasoning: developing learners’ understanding of shapes and geometry in the classroom to promote relevance and engagement. • Hands on activities from Bala Wande involving reasoning about shapes and relationships between shapes	BW Video Group activities Feedback

25 min	Presenter: Tutula Unlocking maths potential with space and shape – Tangrams • Use tangram pieces to create shapes and reason geometrically	Group activities Feedback
15 min	Presenters: Vuyo, Tutula and Ingrid Consolidation and wrap up	Q&A

The activities and worksheets to be used in the workshop

All the activities we will be doing are available through open source and on the Funda Wande website (https://fundawande.org/). We will bring copies of the planned activities and demonstrate how teachers can access the activities for themselves. We will also bring all other resources that are needed for activities.

BUILDING A FOUNDATION BY UNDERSTANDING, COMPARING, AND ORDERING FRACTIONS PRACTICALLY, BY USING VISUAL AND COLORFUL MANIPULATIVES AND INTEGRATING LANGUAGE AND LIFE SKILLS

Mary-Ann Keswa

Kruitberg Primary School, Bloemfontein

INTRODUCTION

Why I chose to talk about fractions:

Fractions are important because they represent parts of a whole. They are important in real life because they help us understand essential tasks like cooking, telling the time, shopping, and even understanding medical prescriptions.

What is a fraction?

- Fractions are defined as the parts of a whole. A whole can be an object or a group of objects.
- Mistakes that learners make when learning fractions:
- Comparing fractions incorrectly. Believing that 1\4 is bigger than 1\2.
- Believing that 4 is bigger than 2 without considering that an object is divided into more pieces or parts.

Why are fractions taught?

Fractions are taught because they are a fundamental mathematical concept, crucial for understanding advanced Math and essential for everyday applications like measuring, cooking, and understanding discounts.

What am I going to do in the talk?

- I am going to present practically and involve students highly and use visual manipulatives such as card board cut into fractions, ruler, colourful ice cream sticks, crayons, magnetic fractions chocolate slabs and also define the meaning of fractions.
- Use different colors and concrete materials to represent fractions. (Integration)
- Use different shapes as well and allow the students to do practical with fractions.
- Teaching mathematical language is key to understanding the topic better and undergoing activities easily.

CONTENT

- Young children are concrete learners. Meaning they need sensory experiences to develop their learning. The more they see and touch Manipulatives, the more they remember the concept and do better in the activity.

- The advantages of using colourful visual manipulatives include allowing learners to move from concrete experiences to abstract reasoning. It helps learners hone their mathematical thinking skills.
- They remember the concept longer and perform better in written and practical activities.
- Seeing and touching the objects introduces them from the unknown to the known.
- Disadvantages of using visual manipulatives: Learners can become too reliant on the object and context.
- They may have difficulty transferring their knowledge to new contexts, different testing formats, or abstract representations**.**

MATHEMATICAL LANGUAGE

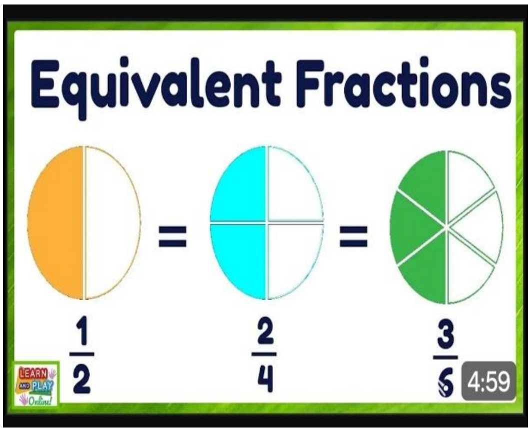
- Mathematical language is part of the language taught daily.
- Integrating Math with other subjects is also important.
- This topic is integrated with Language and Life Skills.
- Different colours are used in the visual manipulatives in order to enhance the lesson because children adore bright colours, and they ace the lesson due to colour coordination.
- An Oral and practical activity is going to be done by students using coloured sticks, magnetic fraction boards, chocolate slabs, and crafted fractions with card boards. This part forms part of Language and Life skills.

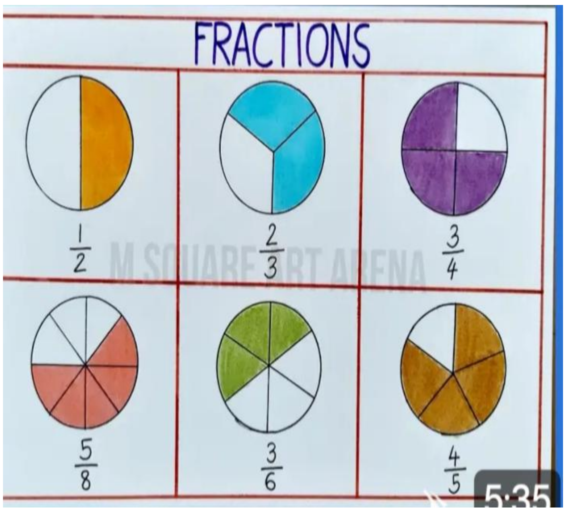
DESCRIPTIONS

Manipulatives

- Concrete objects students can physically interact with to explore and understand abstract concepts.
- A fraction: it's a part of a whole.
- Denominator: The number below the line. The second number in a fraction written with a slash, like ½, indicates the total number of equal parts into which a whole is divided. (divisor)
- Numerator : The number above the line in a vulgar fraction shows how of the parts indicated by the denominator are taken. eg 2/3

Workshop Manipulatives





CONCLUSION

- Making mathematics fun is one of the golden strategies e.g., Celebrating special Math games and playing Math games.Take National and International Mathematics Day into consideration because it enhances the love of Math within schools.
- How we celebrate Math Day at our school…Every grade is allowed to celebrate by doing any mathematical activity suitable for the grade. We do role plays mathematically and play games like snakes and ladder and also play indigenous games (diketo) because it also includes counting, etc.
- Use visual aids (Manipulatives) and picture books.
- Use modern technology: Present fractions online and have learners use tablets in class for practical Lessons.
- Ask interesting questions when busy with lesson presentations. Let learners engage in the form of understanding. Involve all the learners in the lesson despite their academic statuses.
- Take a hands-on approach. Be more present in a lesson and show understanding as a teacher. Learners will show more interest in the lesson as well.
- Encourage cooperative participation by giving them theoretical activities. Create a question-and-answer session.
- Enhance peer assessment whereby learners assess each other’s activities.

TEACHING INTEGERS AND ALGEBRAIC EXPRESSIONS USING ALGEBRA TILES

Fusi Rantene

Free State Department of Education

Abstract

Algebra is a crucial domain in mathematics. It is a generalized version of arithmetic in which letters and signs are employed to achieve the generalization. It is undeniably an abstract domain due to the utilization of letters and signs. Algebra is regarded as a difficult branch of mathematics due to the nature of generalization and abstraction. (Pranada, Dizon & Sabalza, 2019).

The workshop targets grade 8&9 participants who are involved in teaching or supporting teachers. Manipulatives such as algebra tiles will be used to model integers and algebraic expressions. Algebra tiles are rectangular shapes that provide area models of variables and integers.

Participants will be working in pairs to experience hand on activities using Algebra tiles to teach integers and expressions. By the end of the workshop participants will be able to teach addition and subtraction of integers and expand and simplify algebraic expression using algebra tiles. The knowledge gained may be extended further by exploring use of these tiles in teaching algebraic equations.

This workshop will further enhance teacher's knowledge of using algebra tiles as a teaching tool to improve learners' skills (Pranada, Dizon & Sabalza, 2019).

Why is the workshop important?

The "equal sign," "variable," and "unknown" are the fundamental concepts of algebra and the errors learners make in these concepts cause them to struggle with algebra.

Amidu, Salifu and Nyarko (2020:26) found in their research that some teachers find algebra difficult to teach, because it appears less concrete to them. The difficulty stems from working with variables and their notations. Different approaches to teaching algebra are needed to mitigate the challenges and promote conceptual understanding.

The use of algebra tiles might assist teachers to better understand why learners struggle with the concepts of like and /or unlike terms and the workshop might prepare teachers well for manipulation of algebraic expression. The workshop will follow Concrete Pictorial Abstract (CPA) approach. The CPA sequence will benefit teachers so that they help learners learn algebra concepts that they will struggle with if they solely utilize the symbolic stage.

How will it help participants?

The introduction of algebra tiles and other manipulatives into the classroom have a potential to provide participants with great chances to empower learners with a variety of learning styles. Through hands-on activities, this workshop will assist teachers to become familiar with the usage of algebra tiles. Teachers using algebra tiles for the first time will become comfortable with using them in their classrooms. Experienced teachers will learn new applications and extend to other concepts.

Description of content of workshop

This workshop contains activities suitable for use by participants involved in grade 8&9, though they may be extended to grade 10. The approach of the workshop is on utilization of algebra tiles to model integers and expressions as well as use of models to simplify integers and expressions. Each participant will receive a template of algebra tiles to print for learners.

What will be done in the workshop?

In this workshop, the facilitator will model algebraic expressions using tiles on a data projector. The participants will work in pairs to experience the benefits of hands-on activities using a set of algebra tiles to solve activities on integers and algebraic expression such as

- to model integer arithmetic for addition, subtraction,
- Add like terms (tiles) using the rules of integers to get the coefficients. The shape of the tiles determines the type of term x, x^2, or unit.

How will the time slot be broken up?

Item	Duration
The purpose of this workshop is to help you help your students learn how to use algebra tiles effectively.	10 Minutes
Have participants spread the algebra tiles on their worktables and examine them.	15 Minutes
Activity 1: Adding Integers	20 Minutes
Activity 2: Subtracting Integers	20 Minutes
Activity 3: Simplifying Algebraic Expressions	20 Minutes
Activity 4: Adding and Subtracting Polynomials	20 Minutes
Workshop Closing: Recapping the Activities	15 Minutes
	120 Minutes

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AUTOMATICITY IN FOUNDATION PHASE MATHEMATICS

Lerato Mohale

Brebner Primary School, Bloemfontein

Overview

Lerato Mohale is a 28 years old young lady from Brebner Primary School in Bloemfontein under the Mangaung Metropolitan Education District. She holds a B.Ed. Foundation Phase qualification and Honors degree in Psychology of Education. She has taught professionally for 5 years and informally for 4 years which is a total of 9 years of teaching experience. She has taught grade 4-7 Mathematics at Hermana Primary School in Lady brand, Free State in 2021. She then taught grade 3 for 5 years and grade 2 for the past 2 months. Automaticity is the ability of a learner to effortlessly recall a fact. The presentation will be giving skills to educator's skills on how to achieve that for basic facts in addition and subtraction. This is going to help teacher to be able to help learners who have barriers to learning and have not achieved the goals set in the curriculum.

The Objective

> *The presentation will have these objectives which will be fulfilled at the end of the presentation*

What is Automaticity in Mathematics

The Basic Facts in Addition and Subtraction

Drilling bacic facts

Fun activities and games:

Ice cream addition

Jumping math

Subtraction War

Number Puzzle

More or less game

African board 1: No name

<u>Resources to be used;</u>

Marbles

Apples

Large number line

Group of squares

Dice

Crayons
Blank paper
Crayons
Ice cream sheet
Hula hoops
Deck of cards
Timer
Number cards numbered 1-10
36 Number cards numbered 1-18
Marker/stick for marking
The field/classroom

- Conclusion/Summary

Execution Strategy

The methodology of delivering this presentation will be in a form of Demonstration Method where I will do the presentation demonstrating the activities and using the educators themselves to be involved. An hour will be sufficient to do my presentation

Conclusion

I look forward to working with AMESA on equipping educators with best practices in Mathematics. Mathematics is for everyone therefore no once should be excluded from doing and enjoying Mathematics. Learners with challenges should be supported by educators and educators that are equipped can support learners and the parents to bring the best in learners.

I may be contacted at 0678233170 or be emailed at lmohale22@gmail.com. Client's Company

Lerato Mohale

Foundation Phase Educator

HANDS-ON DESMOS WORKSHOP FOR GRADES 7 TO 12

Jenny Campbell, Susan Carletti & Gretel Lampe

The Answer Series

Abstract

To derive maximum benefit from this workshop all participants should have a smartphone or be in the company of another attendee who has one. This workshop is hands-on and is designed to be flexible and adjust to the needs and experience of the participants. Although no prior knowledge of Desmos is required, experienced Desmos users are welcome to attend and to share their knowledge and experience with those being exposed to the software for the first time. The workshop will cover ***Desmos Graphing Calculator*** *and* ***Desmos Geometry****. Technology is part of our everyday lives but is often under-utilized in the classroom. Participants will be able to work at their own pace and level. The nature of the workshop will be collaborative, with presenters and participants pooling their knowledge and expertise and learning from each other.*

Target Audience: Grades 7 to 12
Duration:2 hours
Maximum number of participants: Any number that can be accommodated

Motivation for the Workshop

Desmos is **free software** that can be used on a smartphone or laptop. The app is particularly easy to use and learner friendly. There are many activities that are accessible and easy to use. This could **level the playing fields** between resourced and under resourced schools, providing learners have access to smartphones. The workshop will **equip teachers to use this technology** in the classroom. Although it is easier to demonstrate the software with the aid of a data projector, it is still possible for this software to be used without one. One of the aims of this workshop is to equip teachers to work dynamically with graphs, rather than with pen and paper.

What will be done at the workshop
The focus of this workshop it to expose teachers to Desmos.
All the activities are explorative and are designed to show what is possible.
Teachers will work in small groups and will be able to work at their own pace.

The worksheets are designed to ease the transition into the use of the software.
They are a tool, rather than a must do.
The entire workshop will be hands-on teachers will be actively engaged throughout.

Time

Introduction (30 minutes)
Connecting with participants to ascertain

- the grades/phases they are involved in
- their experience, if any, in the use of Desmos
- their expectations
- making sure everyone can connect to the apps they need
- encouraging everyone to find at least one person they can collaborate with
- make sure everyone can scan the QR codes on the worksheets to access the activities

Graph activities (30 minutes)

Activities are designed to cater for both senior phase and FET teachers … they do not have to be working on the same worksheets, so participants can choose what they want to do.
The goal is to show them what is possible and then encourage them to create their own activities at a level appropriate to the level of their learners.

Discussion (15 minutes)
Get feedback from teachers on

- how they feel about the software
- how user friendly they find the software
- how they feel about exposing their learners to the use of the software
- suggestions on how the software can be used

Geometry activities (30 minutes)

There are activities for both phases and there is some overlap.
Once again, teachers can work at their own pace.
It is possible to work at a very basic level, or to set up dynamic constructions that are more powerful.

Discussion (15 minutes) … like the one above

Worksheets cover the following:
Transformation geometry (translations, reflections, rotations, enlargements, reductions)
Euclidean Geometry (straight lines, triangles, quadrilaterals, circles)
Graphs (straight lines, parabolas, reflections & inverse functions)

The Activities and worksheets that will be used in the workshop
1) Point by point plotting and the basics of straight-line graphs

2) Senior Phase Transformations
3) Dynamic Geometry Constructions
4) Parabolas, straight lines, roots and reflections
5) Functions and Inverses

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HANDS-ON PROBLEM SOLVING FOR GRADE 7 (WITH A SPECIAL FOCUS ON USING A VISUAL APPROACH)

Jenny Campbell, Susan Carletti & Gretel Lampe

The Answer Series

Abstract

The goal of this workshop is to empower teachers to introduce problem-solving into the classroom. The workshop will be hands-on with teachers collaborating with each other as they tackle the problems. The problems can all be solved by using a visual approach and no algebraic knowledge is required. The workshop is designed for Grade 7, but teachers of learners in higher grades are likely to find it valuable as well. Mathematics is so much more than a set of rules and formulae and there is nothing more satisfying than tackling a "challenging" problem and finding a solution that is quite easy to understand. Participants will be given time to attempt the problems on their own and then discuss their solutions. A good problem can often be solved with a variety of approaches, and we can all benefit from sharing our own approach, while being open to alternatives. It is very helpful for us to be able to show learners different approaches in the classroom.

Target Audience: Grades 7 (and 8 & 9)
Duration : 2 hours
Maximum number of participants: Any number that can be accommodated within the allocated venue.

Motivation for the Workshop

Problem solving is often regarded as too difficult for weaker learners and tends to be written off because of lack of time. It can be the stimulus needed to motivate learners to want to understand mathematics better. Maths is so much more than rules and theory and learners who enjoy problem solving are more likely to find themselves passionate about mathematics. None of the questions covered in this workshop require any knowledge of algebra, hence the focus on Grade 7. The problems can, however, be tackled by learners in Grades 8 to 12 as well.

What will be done at the workshop

The focus of this workshop it to expose teachers to problem solving in the absence of algebra, simultaneous equations and other strategies that depend on advanced mathematical knowledge.

Teachers will work in small groups and will be able to work at their own pace.

The entire workshop will be hands-on, with teachers being actively engaged throughout.

Time will be allocated to discussion of different approaches so that we can all learn from each other.

Time

Introduction (10 minutes)

Connecting with attendees to ascertain

- the grades/phases they are involved in
- their experience, if any, in exposing learners to problem solving
- their expectations
- encouraging everyone to find at least one person they can collaborate with

Brief problem-solving discussion (10 minutes)

What strategies can be used to tackle problems that require a visual approach?
Will all visual problems work with the same approach?
What are the kinds of questions we should ask ourselves when attempting these problems?

ACTIVITIES

Questions 1 - 4 (20 minutes)

Allow everyone sufficient time to tackle all these questions before they are discussed.
Ask for volunteers to share their methods.
Discuss alternative solutions.

Questions 5 – 8 (20 minutes)
Invite solutions from teachers …assist with alternative solutions where necessary.

Questions 9 – 12 (20 minutes)
Invite solutions from teachers …assist with alternative solutions where necessary.

Questions 13 – 14 (8 minutes)
Invite solutions from teachers …assist with alternative solutions where necessary.

Questions 15.1 – 15.4 (12 minutes)
Invite solutions from teachers …assist with alternative solutions where necessary.

Questions 16 – 17 (10 minutes)
Invite solutions from teachers …assist with alternative solutions where necessary.
Wrap up & motivation to integrate problem solving into teaching. (10 minutes)
Worksheets
There are 4 pages with a total of 20 problems that all a visual approach.

Teaching Multiplication and Division as Inverse Operations in the Foundation Phase using the Bala Wande Resources

Lorna Sako; Mahlatse Sekgobela; Samantha Morrison

Funda Wande

Abstract

The interrelated relationship between multiplication and division has resulted in these operations being referred to as multiplicative reasoning in some contexts. Multiplication and division have been described in literature as being challenging for teachers to teach and for learners to grasp (Lamon, 2005). It is therefore important that teachers develop a deep conceptual understanding of these operations and how to teach them because this will not only improve their teaching practice, but also increase learners' opportunity to learn. Multiplicative reasoning is considered important in mathematics teaching and learning because it underpins learners' understanding of further number working like ratio, proportion, and percentage.

This workshop aims to address the following questions in a practical way by drawing on the lessons, activities and resources used in the Bala Wande programme:

- *How can I teach multiplication and division in a Foundation Phase classroom in ways that make sense to learners? and*
- *What resources can I use to develop learners' understanding of multiplication and division as inverse operations?.*

Target audience: Foundation Phase teachers
Duration: 2 hours
Maximum no. of participants: 50
Motivation for the workshop: Why is this workshop important, how will it help participants?

The aim is for every teacher who attends the workshop to leave with a better conceptual understanding of the interrelated operations: multiplication and division, and greater knowledge of how to develop Foundation Phase learners' understanding of these inverse operations through the use of sound teaching practices and concrete resources. The workshop will include strategies to build understanding such as repeated addition, grouping and sharing, skip counting and arrays.

The Bala Wande programme is founded on the belief that access to equitable mathematics education is the right of all children in South Africa. Through this workshop, more teachers will have access to equitable teaching and learning opportunities gained from 'best practice'. All the 'best practice' information that will be shared has been gleaned from the research and development done by Bala Wande in South African classrooms. The Bala Wande programme consists of a set of packaged materials which includes a Teacher Guide (TG), which provides day-by-day guidance for the teaching of mathematical concepts, a Learner Activity Book (LAB) with worksheets and games to be used for consolidation of concepts that have been taught in each lesson, and a set of manipulatives for the teacher and learners. Instructional videos also form part of the packaged materials – all of which are aimed at supporting teaching

and learning in the classroom. All Bala Wande materials and resources are freely available on the internet for anyone to download and use.
In this workshop we will draw on Bala Wande's set of open source materials (freely available on the website https://fundawande.org/) to demonstrate how teachers can use concrete materials to teach multiplication in the Foundation phase. We will use practical examples taken from Bala Wande tasks and activities to demonstrate the difference between grouping and sharing division problems. Through modelling and micro-teaching activities – which will involve whole class and group work – we aim to give teachers tangible experience of how to use base-ten blocks as a 'bridge' to support learners' understanding of multiplication. We also aim to give teachers the tools to differentiate between grouping and sharing problems, and how to get learners to distinguish between these problem types for themselves and solve them successfully. Teachers will be shown how to access Bala Wande materials and will be provided with extra resources and tools to use in their classrooms.

Time	Topic	Activity
15 min	Presenter: Mahlatse Ice breaker – Fizz pop Introduction to the workshop & presenters	Participate in Fizz pop doubling/halving Outline of workshop Handouts
20 min	Presenter: Lorna Progression of conceptual understanding about multiplication and division. • Bala Wande- Grouping and sharing activities. • Game: Fair Share	PPT presentation Handouts
30 min	Presenter : Lorna Introducing multiplication as repeated addition. • Activities from Bala Wande: counting in 2s, 3s, 5s • Bala Wande repeated addition activities- Solving word problems	PPT presentation Watch video Group activity -
40 min	Presenter : Samantha How to use arrays to visually represent multiplication and division problems. • Activities from Bala Wande: Rows and columns- multiplication and division number sentences • Game: fast maths with cards- multiply(Bala wande game)	PPT presentation Micro-teaching (groups) Feedback to larger group
15 min	Presenter : Lorna and Mahlatse Consolidation Wrap up	Q&A

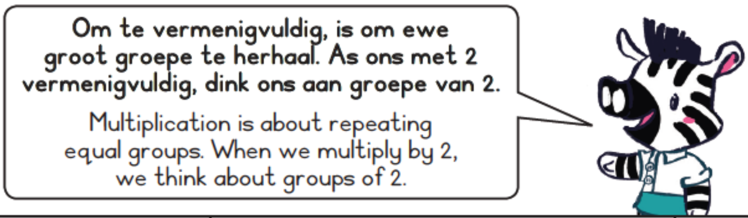
The activities and worksheets to be used in the workshop.
All the activities we propose doing in the workshop are freely available on the Funda Wande website (https://fundawande.org/). We will bring copies of the planned activities for all teachers and will demonstrate how teachers can access the activities for themselves. We will also provide all other resources that are needed for the proposed activities and ensure that there are enough resources to accommodate the participants.

Group Counting

fietse bicycles	1	2	3	4	5	6	7	8	9	10
wiele wheels	2	4	6	8	10	12	14	16	18	20

hande hands	1	2	3	4	5	6	7	8	9	10
vingers fingers	5	10	15	20	25	30	35	40	45	50

Repeated Addition leading to multiplication



3

	Hoeveel kinders is daar? How many children?	6
	Hoeveel oë? How many eyes?	12
	Hoeveel kinders is daar? How many children?	9
	Hoeveel oë? How many eyes?	18
	Hoeveel bottels is daar? How many bottles?	5
2ℓ 2ℓ 2ℓ 2ℓ 2ℓ	Hoeveel liter? How many litres?	10

Multiplication using arrays


DAG 2 • DAY 2
WEEK • WEEK 1
Vermenigvuldig met behulp van rangskikkingsdiagramme
Multiplication using array diagrams

HOOFREKENE MENTAL MATHS | MAAK 20 MAKE 20 → SPELETJIE GAME → KONSEPONTWIKKELING CONCEPT DEVELOPMENT → WERKKAARTE WORKSHEETS

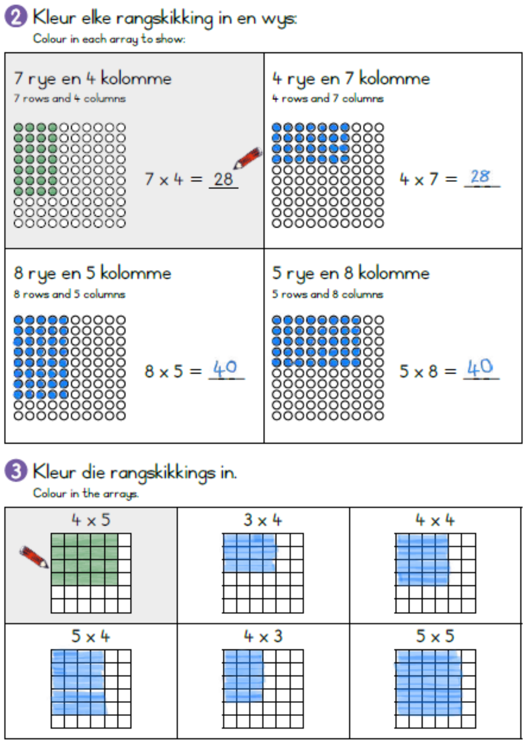
'n Rangskikking is die ordening van voorwerpe in rye en kolomme. Jy kan 'n rangskikking gebruik om te vermenigvuldig! An array is an arrangement of objects in rows and columns. You can use an array to multiply!

1. Tel die aantal rye. Count the number of rows.
2. Tel die aantal kolomme. Count the number of columns.
3. Vermenigvuldig die aantal rye met die aantal kolomme. Multiply the number of rows by the number of columns.

3 rye rows × 4 kolomme columns = 12 produk product

1 Skryf die vermenigvuldigingsin vir elke rangskikking. Write the multiplication sentence for each array

	rye rows 5		rye rows 3
	kolomme columns 3		kolomme columns 5
	vermenigvuldiging multiplication 5 x 3 = 15		vermenigvuldiging multiplication 3 x 5 = 15
	rye rows 2		rye rows 3
	kolomme columns 3		kolomme columns 2
	vermenigvuldiging multiplication 2 x 3 = 6		vermenigvuldiging multiplication 3 x 2 = 6


2 Kleur elke rangskikking in en wys: Colour in each array to show:

7 rye en 4 kolomme 7 rows and 4 columns 7 x 4 = 28	4 rye en 7 kolomme 4 rows and 7 columns 4 x 7 = 28
8 rye en 5 kolomme 8 rows and 5 columns 8 x 5 = 40	5 rye en 8 kolomme 5 rows and 8 columns 5 x 8 = 40

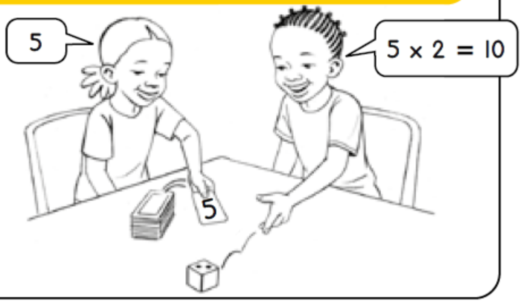
3 Kleur die rangskikkings in. Colour in the arrays.

4 x 5	3 x 4	4 x 4
5 x 4	4 x 3	5 x 5

Multiplication Games

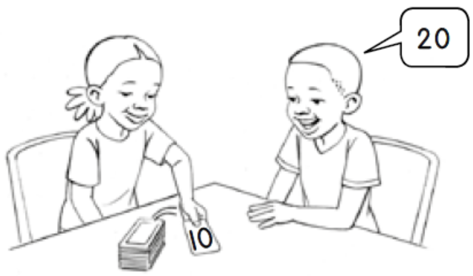
Speletjie: Vinnige wiskunde met dobbelstene en kaarte – vermenigvuldig!
Game: Fast maths with dice and cards – multiply!

- Speel saam in pare. Draai 'n kaart om en gooi die dobbelsteen.
  Play in pairs. Turn a card and throw the dice.
- Vermenigvuldig!
  Multiply!



Speletjie: Vinnige wiskunde met kaarte – verdubbel
Game: Fast maths with cards – double

- Sit getalkaarte 0 tot 20 op 'n hopie neer.
  Place number cards 0 to 20 in a pile.
- Draai een kaart om.
  Flip over one card.
- Verdubbel die getal!
  Double!



BRIDGING THE GAP: ENSURING A SMOOTH TRANSITION FROM SENIOR PHASE TO FET MATHEMATICS

Koketso Lekgwathi , Christiano Mthethwa

Olico Maths Education

Abstract

The workshop enables educators to develop skills that ensure learners proceed smoothly from Senior Phase to FET Phase mathematics education. The workshop combines activities together with games and explanatory sessions about Geometry principles (coordinate geometry, transformation and properties) as well as Functions alongside their algebraic connections. The workshop combines experiential learning with group dialogue to help teachers understand ways of connecting math concepts between Senior Phase and FET Phase while deepening their grasp of Euclidean geometry, algebra and Functions accompanied by practical teaching tools and activities. Teachers should gain abilities to help students connect mathematical concepts together and learn problem-solving approaches. The workshop experience will boost educator instructional effectiveness regarding FET Phase mathematics which will drive better student results while creating solid academic foundations for future learning. The combination of these materials adapted by external mathematics educators and within the OLICO Maths Education program have shown a gradual improvement on learner knowledge and understanding.

Motivation

When students move from the Senior Phase to the FET Phase, they often struggle with a big mathematics problem: they cannot relate different math topics to each other. This makes it hard for them to understand and make sense of math concepts.

Algebraic expressions are a fundamental component of mathematics education, playing a vital role in the development of mathematical understanding and problem-solving skills. (Egodawatt, 2011, as cited in Musi, 2022). To succeed in mathematics, learners must possess a strong foundation in reading, writing, and manipulating algebraic expressions, as well as proficiency in performing calculations and operations involving these expressions. (Ferretti, 2020, as cited in Musi, 2022).

For example, geometry and algebra are closely connected. To solve geometry problems, students need to use algebraic skills. But many students don't see this connection.

This can cause big problems, like students losing interest in math, feeling anxious about mathematics, and lacking confidence in their mathematical abilities.

We need to help students to see how math topics fit together. This workshop focuses on the importance of integrating various math topics to foster a more comprehensive and nuanced understanding of mathematical concepts. We will explore innovative strategies for highlighting connections between disciplines, ultimately enhancing students' mathematical proficiency, critical thinking, and problem-solving skills.

Content Description

1. **Welcome and introduction to the workshop:** workshop including a demonstration of icebreakers and energisers that can be used with learners to foster fluency in core mathematics facts.

[15 Minutes]

2. **Relating the sketching of graphs to algebra:** In this section we will show step-by-step procedures for sketching two functions on a single Cartesian plane. Teaching staff should utilize their expertise in factorization then substitute values and solve simultaneous equations to understand the concept. The algebra skills enable us to draw graphs while demonstrating the connection between algebra and graphical representation.

[20 minutes]

3. Given functions $f(x) = -x^2 + 9$ and $g(x) = -3x + 9$

3.1 Draw **f** and **g** on the same system of axes.

3.2 Use your graphs to determine for which values of x:

3.2.1 $f(x) = g(x)$ 3.2.2 $f(x) > 0$

3.3 How does the graph of $h(x) = -x^2 - 9$ compare with the graph of $f(x)$?

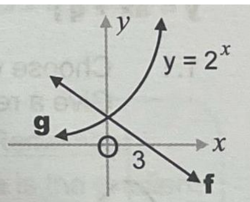
3.4 Give the equation of the reflection of **f** in the x-axis.

(Acknowledgement: A. Eadie and G. Lampe Grade 10 Mathematics 3-in-1 CAPS – The Answer Series page 6.23)

2.6.1 The point of intersection of **f** and **g** is ....

2.6.2 The equation of this straight line graph, **f**, is ....

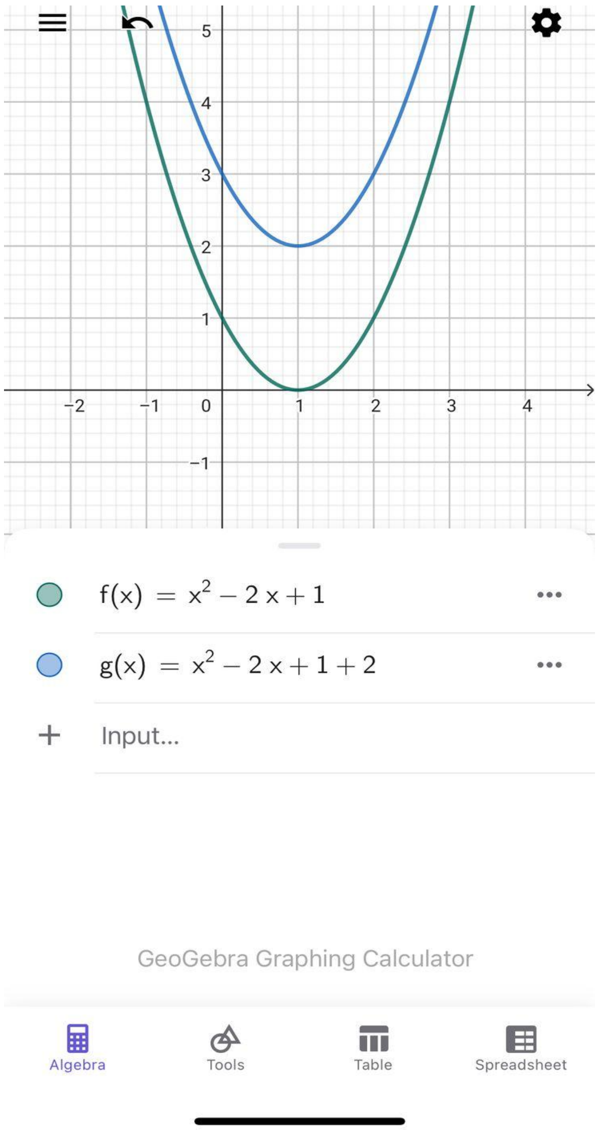
2.6.3 The range of **g** is ....



(Acknowledgement: A. Eadie and G. Lampe, Grade 10 Mathematics 3-in-1 CAPS – The Answer Series page 6.25)

3. **Discussion of Transformations with the use of GeoGebra (for visualisation):** In this section, we will demonstrate the importance of integrating technology into the educational space, where we focus on how the changes made to the equations affect the shape and position of the graph as well as the classification of the changes. The participants will also have an interactive experience of using GeoGebra.

[20 minutes]



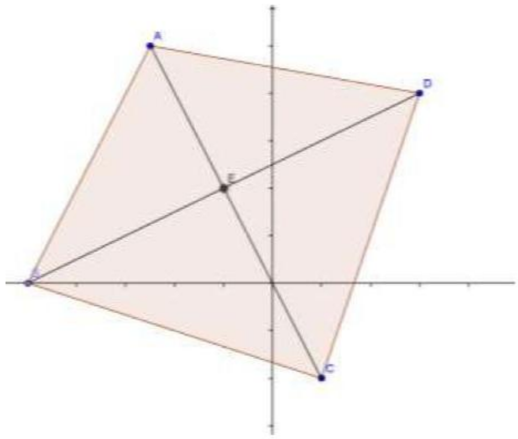
4. Game: **Online-** Kahoot (Quiz on Parts of topics that we have engaged on)

[15 minutes]

5. **Group Activity: Application of algebraic techniques into Analytical geometry:** For this part of the presentation, participants will be divided into groups and presented with questions that require collaborative problem-solving. This promotes peer-to-peer learning through the THINK-PAIR-SHARE approach. The main objective is to review and apply prior knowledge of algebra to analytical geometry properties (e.g., perpendicular lines, distance formula), and promote collaboration and communication among group members. This activity encourages active learning, critical thinking, and teamwork which are also important skills needed in a diverse classroom.

[20 minutes]

Given the following quadrilateral with coordinates ABCD. Given that A is (-2.5; 5), B is (-5; 0), D is (3; 4).



a) Determine the midpoint E of BD. (R)

b) Determine the length of AB. (R)

c) Prove that AEC is perpendicular to BED. (P)

d) Using Pythagorus, determine the length of AE. (C)

e) Determine the area of triangle ABE. (C)

f) Determine the length of AC given that the length of DC is $2\sqrt{10}$. (P)

g) Determine the coordinates of C, given that the line through DC passes through the y-axis at -5. (P)

h) Determine the length of AD and BC. (R)

i) What kind of geometric shape is ABCD? Give 2 reasons for your answer. (P)

(Acknowledgement: SHARP WORKSHEET – Grade 10 Analytical geometry)

6. GAME:

- **Offline -** 30 seconds (general knowledge of Maths terms)

[15 minutes]

1. Triangle 2. Sphere 3. 45° 4. Perpendicular bisector 5. Coefficient	30 seconds Game OLICO FET
1. Quadrilateral 2. Multiple 3. Mode 4. 180° 5. Chord	30 seconds Game OLICO FET
1. Angle 2. Parallelogram 3. Cylinder 4. Factor 5. Trinomial	30 seconds Game OLICO FET
1. Rhombus 2. Cube 3. Rational number 4. Variable 5. Sine	30 seconds Game OLICO FET
1. Equation 2. Median 3. Square 4. Irrational number 5. x-intercept	30 seconds Game OLICO FET

7. Q & A - The end

[15 minutes]

Technical Requirements

A fully equipped lab with at least 50 computers or cell phones, a projector, audio and an internet connection.

Conclusion:

In conclusion, the integration of mathematical concepts is a crucial aspect of success in FET Maths. As learners progress, it is essential to recall and build upon the fundamental skills acquired in previous grades. By reinforcing their mathematical foundation and embracing a growth mindset, learners can develop a deeper understanding of complex concepts and cultivate the confidence to tackle challenging problems. Ultimately, this integrated approach will empower learners to navigate the demands of FET Maths with ease and lay the groundwork for future academic success.

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EQUAL EXCHANGE: EFFECTIVE TEACHING OF PLACE VALUE IN THE FOUNDATION PHASE

Thobeka Ndamase; Reneilwe Matjutla

Funda Wande

Abstract

Place value is a fundamental concept in understanding numbers and an important building block for understanding operations with multidigit numbers (Houdement & Tempier, 2019). The mastery of place value is crucial for future academic success because children deal with bigger number ranges as they progress through school. However, research has shown that many learners in the Foundation Phase (FP) struggle to grasp Equal Exchange, which underpins the concept of Place Value (Bynner & Parsons, 1997).

The workshop

Target audience: Foundation Phase teachers
Duration: 2 hours
Maximum no. of participants: 50
Motivation for the workshop: Why is this workshop important, how will it help participants?

By presenting this workshop on a keynote topic for early grade mathematics teaching and learning, we aim to provide teachers with access to relevant and equitable mathematics education. We will address Equal Exchange and how it relates to Place Value by showing teachers very practical ideas that they can use to develop learners' understanding of Equal Exchange. Bala Wande materials will be used to demonstrate how teaching Place Value to FP learners, with an emphasis on Equal Exchange, can be effective in developing deep conceptual understanding of Place Value. By focusing on Equal Exchange involving real-life scenarios and manipulatives, we aim to provide teachers with a more robust understanding of this concept as well as an understanding of the connections and relationships among the related concepts.

The Bala Wande programme is founded on the belief that access to equitable mathematics education is the right of all children in South Africa. Through the combination of exciting presentations, lively group discussions, and fun practical activities, participants will gain a hands-on understanding of how to use Equal Exchange to support learners' understanding and application of Place Value. By using practical examples from Bala Wande tasks and activities, we will demonstrate how common problems with regard to implementing the column method can be resolved. Our goal is to provide teachers with direct experience in using base-ten blocks as a tool to enhance learners' understanding of Place Value. Teachers will be shown how to access Bala Wande resources and will receive additional materials and tools for use in their classrooms.

Time	Topic	Activity
15 min	Presenter: Thobeka Ice breaker – Fizz pop Introduction to the workshop & presenters	Participation Outline of workshop Handouts

20 min	Presenter: Reneilwe Game: How many 10s, how many 1s	PPT presentation Paired activity
30 min	Presenter: Thobeka Video on Equal Exchange Equal exchange Activity	PPT presentation Watch video Group activity -
40 min	Presenter: Reneilwe Place value activity using the column method (The use of base 10 blocks)	PPT presentation Micro-teaching (groups) Feedback to larger group
15 min	Presenter: Thobeka Consolidation Wrap up	Q&A

The activities and worksheets to be used in the workshop

All the activities we propose doing in the workshop are freely available on the Funda Wande website (https://fundawande.org/). We will bring copies of the planned activities for all teachers and will demonstrate how teachers can access the activities for themselves. We will also provide all other resources that are needed for the proposed activities and ensure that there are enough resources to accommodate the participants.

References

Houdement, C., & Tempier, F. (2019). Understanding place value with numeration units. *ZDM*, *51*, 25-37.

Bynner, J., & Parsons, S. (1997). *Does Numeracy Matter? Evidence from the National Child Development Study on the Impact of Poor Numeracy on Adult Life*. Basic Skills Agency, Commonwealth House, 1-19 New Oxford Street, London WC1A 1NU, England, United Kingdom (6.50 British pounds).

USEFUL STRATEGIES FOR TEACHING SUBTRACTION IN THE FOUNDATION PHASE

Ntombikayise Nkcithakala, Sarel Mahlahlane

Olico Maths Education

Abstract

In our Olico Maths Education after-school Maths Clubs, we aim to provide fun, exciting and engaging Maths activities for learners in Grades R-6. The lesson plans and activities for the Maths Clubs are designed to build strong number sense skills and a love for Mathematics.

Subtraction is often a challenging concept for young learners, with many finding it harder to understand than addition. This workshop will equip educators and facilitators with practical and engaging strategies for teaching subtraction in the Foundation Phase. Through energising warm-ups, activities and games, participants will experience hands-on approaches to build their learners' confidence and fluency in subtraction. Our goal is for participants to leave the workshop motivated and equipped with valuable strategies they could incorporate into their classrooms to make subtraction a fun and accessible concept for learners.

INTENDED AUDIENCE

Foundation Phase (1-3) teachers and facilitators looking for new and practical ways to teach subtraction.

DURATION

The workshop is 2 hours long.

MAXIMUM NUMBER OF PARTICIPANTS

The workshop caters for up to 30 participants.

MOTIVATION FOR THE WORKSHOP

Our experience working with learners in the Foundation Phase has shown that many struggle to understand and work with subtraction. Learners are often frustrated and some are even confused when they do subtraction. Few Foundation Phase learners know how addition and subtraction are related.

As an organisation, we recognise the importance of building a strong foundation in Maths from an early grade. Teaching subtraction can be a daunting task, especially when learners lack confidence or struggle to understand the concept. Our workshop aims to address this challenge by sharing interactive and practical strategies for teaching subtraction. We will showcase approaches that have proven effective in our work with learners. By equipping participants with innovative teaching methods and resources, we hope to empower young learners to develop a deeper understanding and love for mathematics.

DESCRIPTION OF WORKSHOP

***Introduction* (15 min)**

In this session, participants will reflect and talk about their experience teaching subtraction to learners. Share challenges they have come across and strategies they have used to make subtraction easier.

At the end of the session, participants should:

- Be able to reflect on their subtraction teaching methods and successes
- Be able to identify key challenges in learners' understanding of subtraction
- Discover that subtraction can be taught and learned differently

***Warm-ups, Activities and Games* (90 min)**

The workshop requires engagement and participation. It will involve group and pair work/discussions on subtraction activities and games. Participants can provide feedback and ask questions on each part of the session.

- Warm-up: *Shakedown*
- Activity: *Story sums using the Part-Part Whole diagram*
- Game: *Fact Family game*
- Warm-up: *Fizz pop back to 10*
- Activity: *Bridging 10 subtraction*
- Game: *Dice and card subtraction*
- Warm-up: *What number is in my head?*
- Game: *Flower visitors backwards and forward*
- Game: *Salute*

***DISCUSSION* (15 min)**

Participants give reflections and feedback on the workshop.

REQUIREMENTS: A projector, audio and internet connection

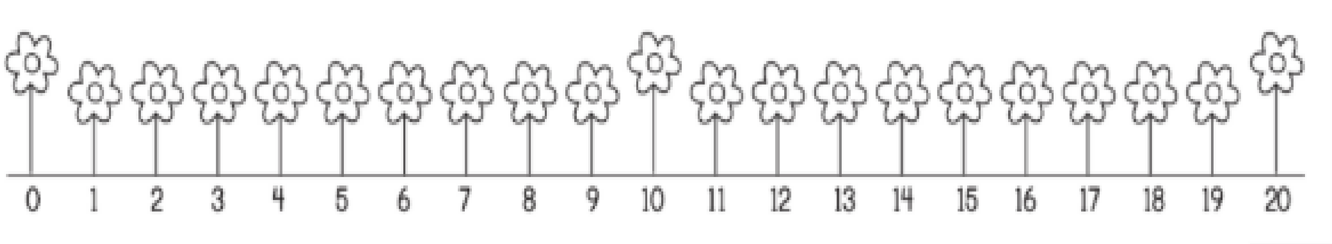
Part Part Whole (PPW)

FACT FAMILY GAME

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

Write an addition or subtraction fact for each triple you choose.

FLOWER NUMBER LINE



INTEGRATING "KNOW AND SHOW" CHARTS INTO ANALYSIS GEOMETRY FOR GRADES 10-12 EDUCATORS

Victor Mbhiza

Axium Education

Abstract

*In this interactive workshop, mathematics educators will learn to use the "**Know and Show" charts** in conjunction with the " **I Do, We do", You Do"** and " **See It, Name It, Do It"** approaches to teach complex concepts in analytical geometry. Topics such as properties of polygons, collinearity, Euclidean and analytical geometry integration, and tangent length calculation will be explored. Participants will experience a structured, HANDS-on learning environment where they gradually gain understanding, engage in discussion, and apply their knowledge through independent problem solving problem-solving. This workshop will provide educators with practical strategies and resources to improve their teaching methods and help learners master abstract geometric concepts*

Target Audience: Grade 10 -12 Mathematics Educators

Duration: 2 hours

Maximum Number of Participants: 30- 40 Participants

1. Motivation for the Workshop

Mathematics educators often face challenges in teaching analytic geometry concepts that are abstract, such as the properties of polygons, collinearity, and tangent lengths. The 'know and show 'charts method combined with the "I **Do, We Do and You Do "approach** and the "**See it, Name it and Do it**" approach offers a structured, dynamic way of teaching these concepts.

- "**See It** "helps participants to observe the theory and the "what" of a concept.
- "**Name It**" encourages the identification and deepening of comprehension of the material.

- "**Do It**" empowers participants to apply the concept in practice.

This workshop will provide educators with tools to help learners understand complex geometry concepts step -by -step while fostering active participation.

2. Description of Content of the Workshop

Overview:

The session will combine the "**See It, Name It, Do it**" approach and "**I Do, We do, You Do**" teaching strategy. Through these methods, participants will explore key concepts in analytical geometry, such as:

- **Properties of polygons:** Proving properties of polygons using analytic geometry.
- **Collinearity**: Understanding and proving collinearity using coordinates and equations.
- **Integration of Euclidean Geometry and Analytic Geometry:** Applying Euclidean geometry axioms and theorems in analytical geometry.
- **Tangent length calculation**: using analytic geometry to calculate the length of a tangent from an external point of a circle.

Each of these topics will be explored using the **"See It, Name It, and Do It"** approach, enhancing both understanding and practical application.

3.0 Time breakdown with " I Do, We Do, You Do" and " See It, Name It and Do It" Approach:

3.1 Introduction and Overview (15 minutes):

- Brief introduction to the importance of the "**Know and show" chart,** " **I Do, We do, You Do",** and "**See It, Name It, Do It"** approaches.
- Explanation of how these approaches will be used in the session to promote both understanding and practical applications.

3.2 **Activity 1: "See It"- Introduction to Properties of polygons (15 minutes):**

- **See It:** The facilitator will present the theory behind the properties of polygons, specifically using analytical geometry to prove side lengths, angles, and symmetry. The key concepts, formulas and theorems will be explained.
- Participants will observe how the "**Know and Show " charts** can be used to describe and demonstrate these properties.
- **Key Focus:** Introduce core concepts that learners need to understand in order to prove properties of polygons analytically.

3.3 **Activity 2: "Name It"- Exploring Properties of Polygons (20 minutes):**

- **Name It**: Participants will:
- engage in a group discussion to deepen their understanding of the materials introduced in the "See It" phase.
- Discuss the various methods and strategies used to prove properties of polygons analytically.
- **Group Activity:** Discuss how to use analytical geometry to prove the properties of different polygons (e.g., parallelograms, rectangles, etc.) through practical examples.
- **Key Focus:** Collaborative discussion to explore different methods and ensure participants fully grasp the concepts.

3.4 **Activity 3: "Do it"- Proving Properties of polygons (25 minutes):**

Do it: participants will:

- Apply the concepts learned during the “See It” and Name It” phases by working independently or in small groups to solve problems related to the properties of polygons.
- use the “know and Show” chart to give “**I Do, We Do”** and “**You Do”** approaches to ensure they can independently apply the methods.
- The facilitator will circulate to provide support as needed.
- **Key focus**: Hands-on application of knowledge to reinforce learning and ensure understanding.

3.5 **Activity 4: “I Do”- Demonstrating Collinearity using Analytical methods (15 minutes):**

- **I Do**: The facilitator facilitates the demonstration of how to prove collinearity of points using the slope formula and distance formula.
- **See It**: the facilitator guides participants to explain the theory behind collinearity and the method to solve such problems.
- **Key Focus:** provide a clear, step-by-step explanation of how to determine if points are collinear using analytical methods.

3.6 **Activity 5: “ We Do”- Solving Collinearity Problems Together (25 minutes):**

We Do: In small groups , participants will

- work together to solve a problem involving collinearity using the “Know and Show” chart.
- apply the methods demonstrated in the “I Do” phase, with the facilitator providing guidance as necessary.

- **Key Focus**: Collaborative problem solving to help reinforce understanding and ensure that participants can apply the theory to real problems.

3.7 **Activity 6: " You Do" - Proving Collinearity Independently (15 minutes):**

You Do:

- participants will reflect individually and solve a problem on collinearity, applying the methods learned and practiced in the previous activities.
- The facilitator will provide individual support where necessary, but participants will solve the problems on their own.
- **Key Focus:** Independent problem -solving to build confidence and reinforce understanding of the concept.

3.8 **Wrap- Up and Reflection (10 minutes):**

Name It: Participants will:

- Reflect on the "See It, Name It, Do It" and " I Do, We Do, You Do " approaches used during the workshop.
- Summarize key takeaways and answer any final questions.
- Share how they plan to implement these strategies in their classrooms.
- **Key Focus:** Reflection and Feedback on how the strategies can be applied in the classroom.

4. The Activities and Worksheets to be Used in the workshop

- **Worksheet 1: Proving Properties of Polygons Using Analytic Geometry**

- This worksheet will guide participants through the **" See It", "Name It"** and **"Do It"** steps for solving problems related to the polygons. It will help participants apply analytic geometry to prove properties such as side length, symmetry and angles.

- **Worksheet 2 Collinearity of Points**

  This worksheet will focus on proving collinearity using analytic methods (e.g., gradient or distance). It will include exercises where participants can work through examples following the "**See It", "Name It",** and "**Do It**" approach.

- **Worksheet 3: Tangent length Calculation**

  A practical worksheet for calculating the length of a tangent from an external point to a circle. This will follow the "**See It", "Name It"** and " **Do It**" steps, allowing participants to gradually master the process.

- **Worksheet 4: Integrating Euclidean and Analytical Geometry**

  This worksheet will guide participants through problems that combine Euclidean geometry axioms and analytical geometry methods, following the " **See It", Name It**", and " **Do It**" approach.

Geometric Constructions

Manare Setati

University of Limpopo

Abstract

Geometric constructions form very important part of mathematics development and is part of the South Africa mathematics curriculum particularly from senior phase (grades 7 – 9) and beyond. Geometric constructions is however, the most neglected content. This is not only because it is a difficult content to teach, but teachers find it difficult to understand. The purpose of this workshop is to provide teachers with the opportunity to engage with geometric constructions as learners of mathematics so that they can in turn know what it takes to engage with the content thereby developing content knowledge for teaching and confidence thereof. The workshop intends to take you through hands on experiences of constructing a give line segment, angle and triangle; construction triangle given three lengths and also an include angle; perpendicular line and perpendicular from a point to a line. Participants will also get experience of using pair of compasses to bisect line and angle. Participants will explore all these by using pair of compasses and straight edge. It is hoped that the experience will provide do with take to school kind of activities that can be used in you grades 7 to 9 classes.

GEOMETRIC CONSTRUCTIONS

Geometric constructions form part of the curriculum for senior phase and yet they are seldom not taught, not because there is no time to teach, but because teachers also find them difficult. You will note that at some point geometry was an optional content area in Further Education and Training (FET) band and most schools opted not to include it as part of the FET mathematics curriculum. As a result, teachers teaching in the senior phase opted not to teach particularly geometric constructions as they believed that it was not useful content area going further. The was despite geometry being import foundational knowledge for learners who were intending to study engineering and science in their post matric journey. This option has since been cancelled and geometry is not compulsory content are for mathematics until grade 12.The above scenario created a gab of knowledge of teaching geometric construction not only because it is difficult to teach but because teachers also have little experience with geometric construction.

Target audience: Senior Phase (Grades 7 – 9)

Duration: 2 hours

Maximum number of participants: 50

How will it help participants?

Van Hiele (1999) in his theory of development of spatial sense wrote "*I believe that development is more dependent on instruction than on age or biological maturation and that* *types of instructional experiences can foster, or impede, development*". I also believe that that teachers need this experience for them to be teaching geometric construction with confidence and better understanding. For teachers to develop knowledge of teaching geometric constructions, they themselves need the experience of engaging in geometric constructions.

Description of content of workshop

The workshop will include **using pair of compasses and straight edge** only:

- Vesica Piscis – the mother of all geometric construction. This will include exploring construction a line perpendicular to another and also construction of an equilateral triangle,
- Copying a given line segment, angle and triangle
- Exploring congruency
- Bisecting a line and an angle

What will be done in the workshop?

All the content will be explored by using pair compasses and straight edge (ruler). Experience has taught us that the use of pair of compasses is difficult for learner and teachers. The experience that they will get during this 2-hour session will develop awareness of the necessity to practice the constructions thereby encouraging teachers to see the need to provide similar experiences to learners.

How will the time slot be broken up?

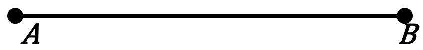
<table>
<tr><th>ACTIVITY</th><th>DURATION</th></tr>
<tr><td>Introduction</td><td>5 minutes</td></tr>
<tr><td>Brief presentation of the van Hiele theory of spatial sense:</td><td>15 minutes</td></tr>
<tr><td>• Vesica Piscis.
• Construction of circles on circumference of other circles</td><td rowspan="2">15 minutes</td></tr>
<tr><td>Exploring Various shapes that can be drawn from construction of circles</td></tr>
<tr><td>Construct a congruent copy of a given:
• line segment,
• angle and
• a given triangle</td><td>15 minutes</td></tr>
<tr><td>Construction of triangle given:
• Three lengths
• An included angle</td><td>15 minutes</td></tr>
<tr><td>Use compasses to bisect a line</td><td>10 minutes</td></tr>
<tr><td>Use compasses to bisect an angle</td><td>10 minutes</td></tr>
</table>

• Construct a perpendicular line, • Construct a perpendicular from a point to a line	15 minutes
Discussions	20 minutes

Introduction:

Activity 1: Construct a congruent copy of a given line segment

Construct a congruent copy of line segment $\overline{AB}$,

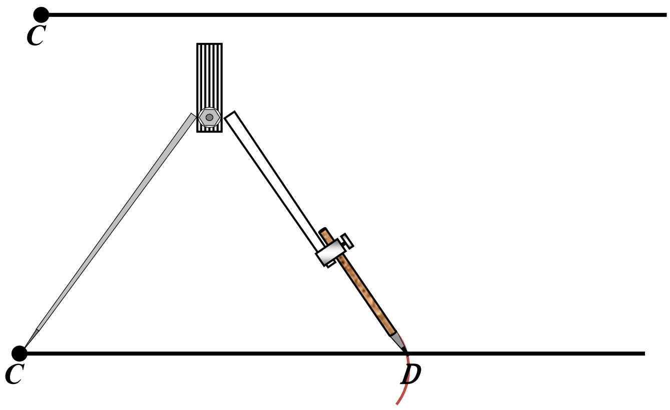


Procedure:

1. With a straightedge draw a segment longer than $\overline{AB}$. Label one endpoint *C*.

2. Open your compass to the length of $\overline{AB}$. With the compass point on C, draw an arc intersecting the segment.

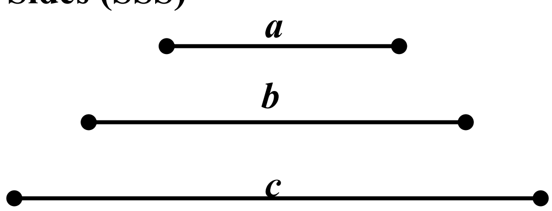
3. Label the point of intersection D. $\overline{AB} \equiv \overline{CD}$.



Discussion: What is the importance of straight edge and compass?

Activity 2: Construct a Triangle Given Three Sides (SSS)

Construct a triangle having sides that measure *a*, *b*, and *c*.



Procedure:

1. With a straight edge, draw a segment longer than the longest segment given
2. On that segment, construct a copy of the segment whose length is *c*. Label the segment $\overline{AB}$.
3. Set your compass for length *b*. With compass point on A, draw an arc above $\overline{AB}$. (This arc is part of a circle with center A and radius *b*.)
4. Set your compass for the length *a*. With the compass point on *B*, draw an arc above $\overline{AB}$ that intersects the arc drawn in Step 3.
5. Point ***C***, where the two arcs intersect, is the third vertex of the triangle. With a straightedge, draw $\overline{AC}$ and $\overline{BC}$.

Investigation 1: The Side Side Side Property

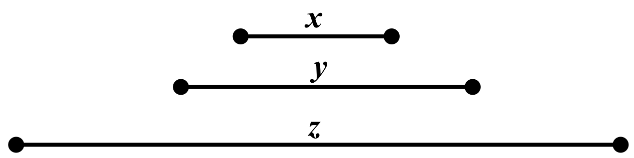
1. Repeat the construction above, but in step 3 put your compass point on B and in step 4 put your compass point on A.
2. Again, repeat the construction in Activity 2 but this time draw the arcs in step 3 and 4 below $\overline{AB}$ instead of above it.
3. Compare the two triangles just constructed. How do they differ? How are they similar? Are they congruent?

Conjecture A: If three sides of one triangle are congruent to the corresponding sides of another triangle, the triangles are congruent.

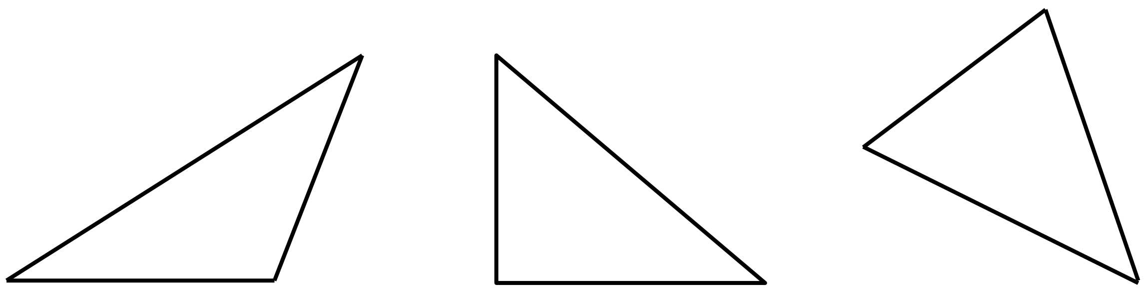
4. Did constructing the sides in a different order change the size and shape of the triangle?

Investigation 2: Can we always construct a triangle from three given lengths?

1. Try to construct a triangle with sides of length x, y, and z, as shown

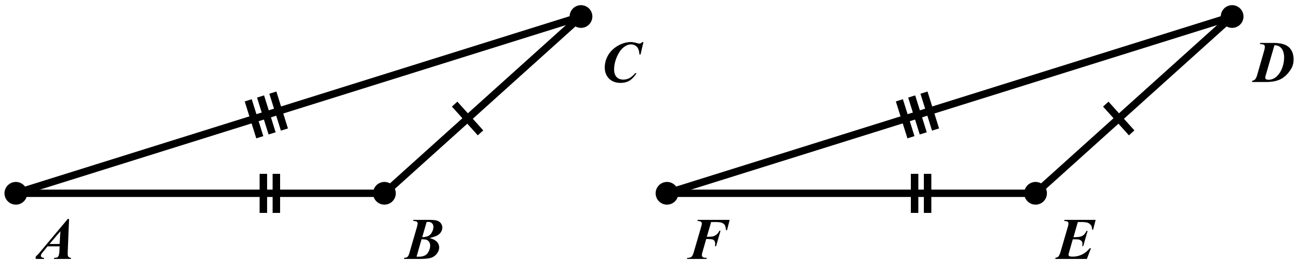


2. Measure the three sides of each triangle shown below. For each triangle, compare the length of each side to the sum of the lengths of the other two sides.



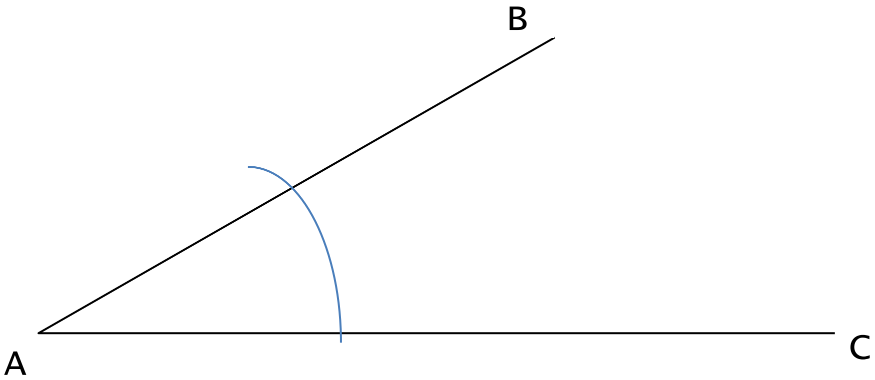
3. What have we learnt? What does **The Triangle Inequality Property** say?

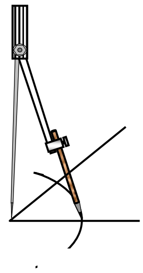
4. Explain why the following two triangles are congruent?



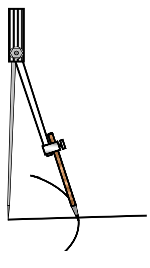
Activity 3: Construct an angle equal to a given angle.

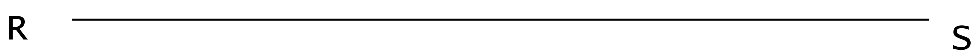
1. Put the anchor point of your compasses on the point A below and draw an arc with radius at least 5 cm that cuts AB and AC. Label the intersection with AB as P and the intersection with AC as Q.





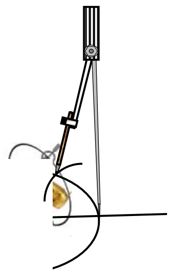
2. Put the anchor point of your compasses on the point R below and draw an arc with the same radius than the arc APQ that you drew in question 1. Label the intersection with RS as T.





3. Set your compasses so that it will draw an arc or circle with radius equal to the length QP in your figure for question 1.

Keep your compasses on radius length QP, put the anchor point



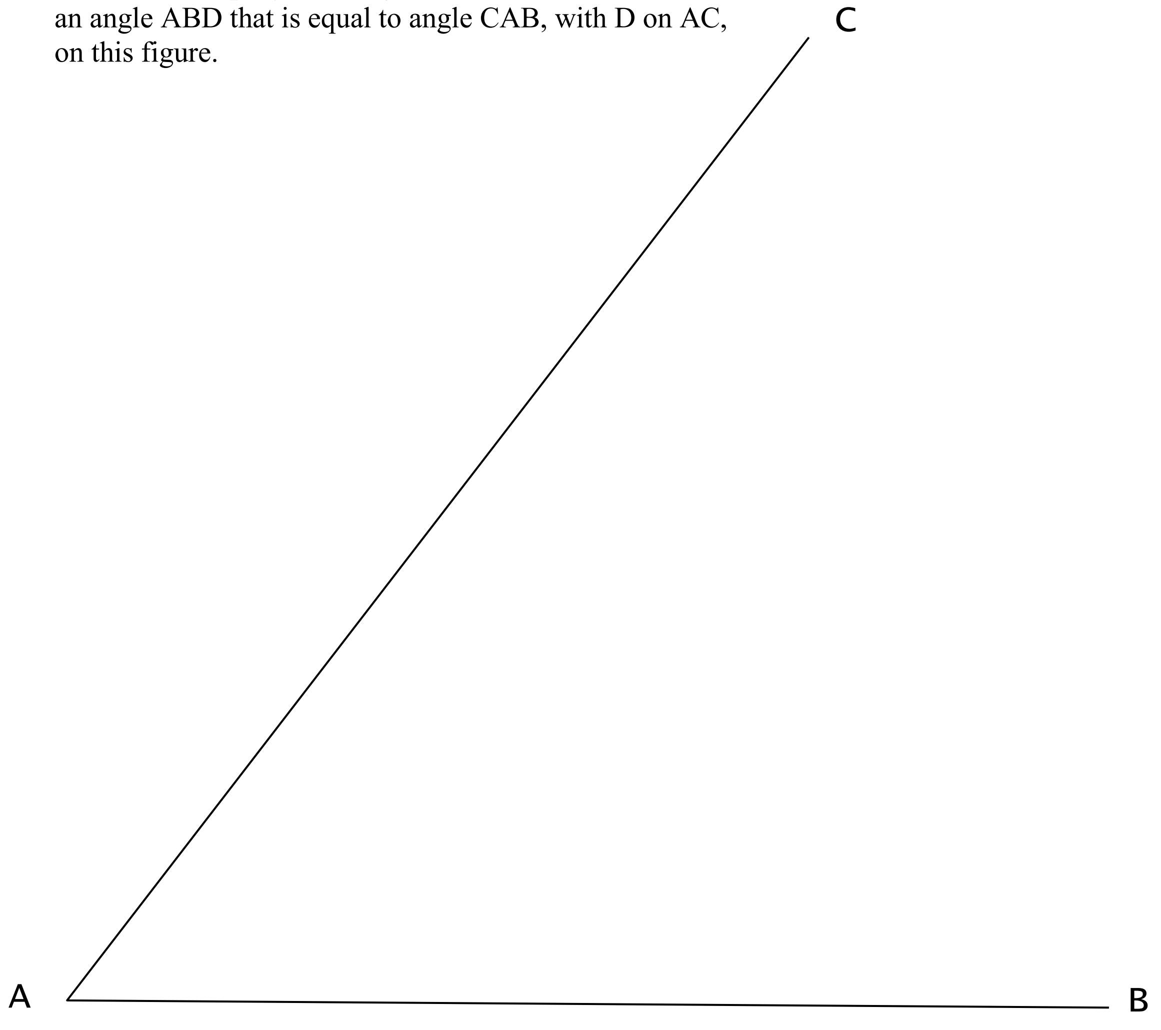
on T in the second sketch, and draw an arc with radius length QP so that it intersects the arc that you have drawn in question 2.

Label the point where the two arcs intersect as U.

Check whether UT = QP (if not, you have made a mistake somewhere).

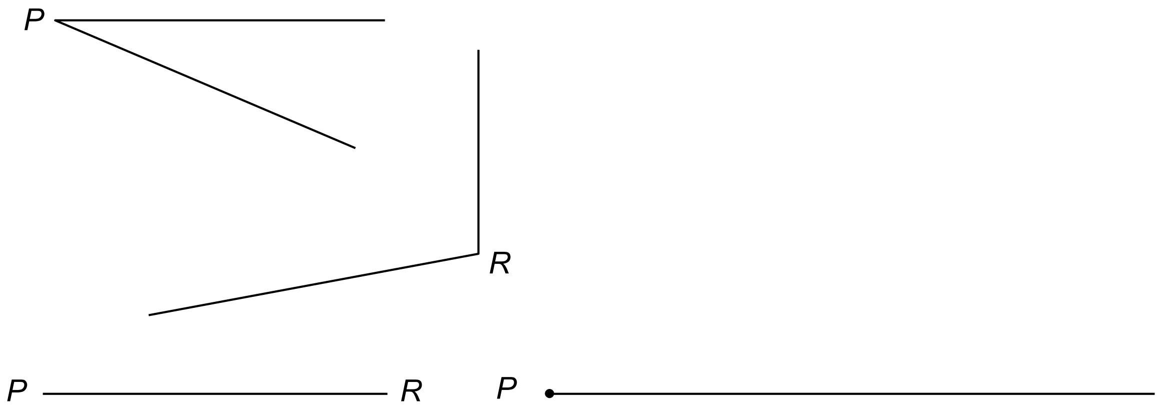
4. Join R and U. Measure angles BAC and URS with your protractor. They should be equal.

5. Use the technique you have just learnt to construct an angle ABD that is equal to angle CAB, with D on AC, on this figure.

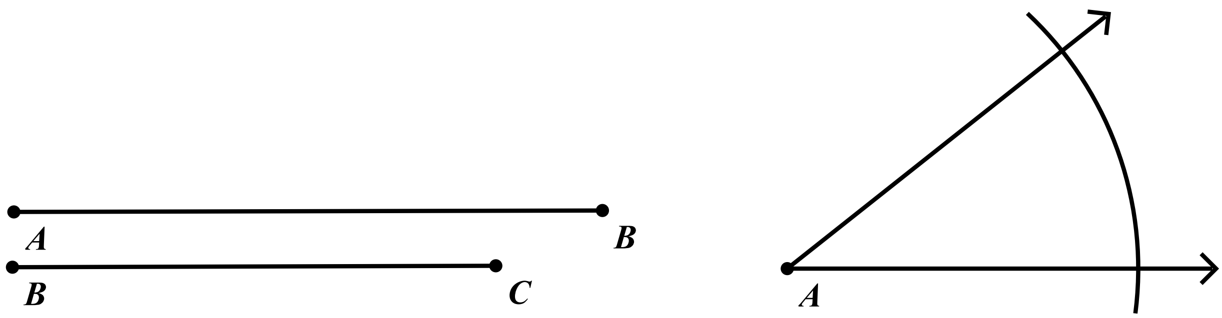


Activity 4: Construct a triangle given an included angle

1. Construct $\triangle PQR$.

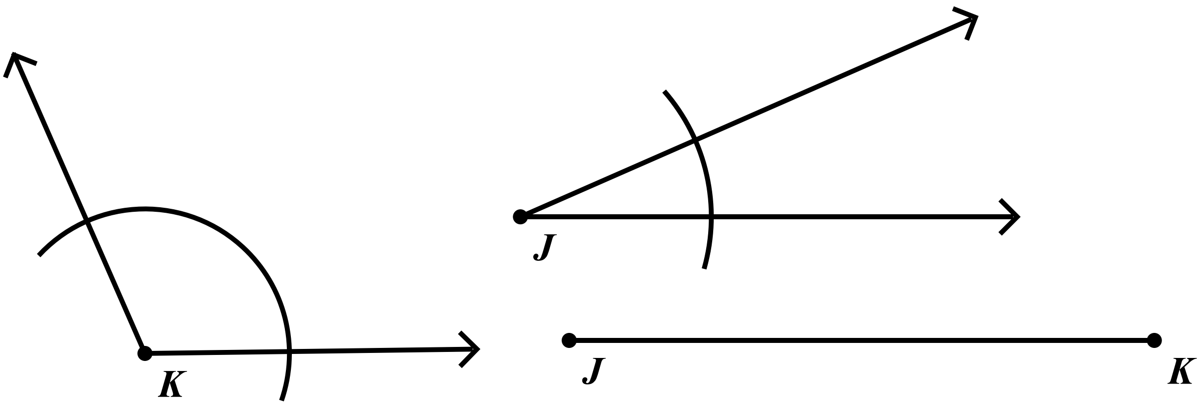


2. Construct two different triangles named *ABC* that have the three given parts.



3. Given:

Construct: □*JFK*



Activity 5: Use compasses to bisect a line

1. Set the compasses wider than half the length of the line segment AB below. Then put the anchor point of your compasses on A below, and draw a wide arc that goes above and below the line AB.
2. Keep your compasses on the same setting, put the anchor point on B, and draw an arc that intersects your first arc in two places. Label the points of intersection of the two arcs as P and Q.
3. Join P and Q. Label the point where PQ cuts AB as T.
4. Measure AT and BT.

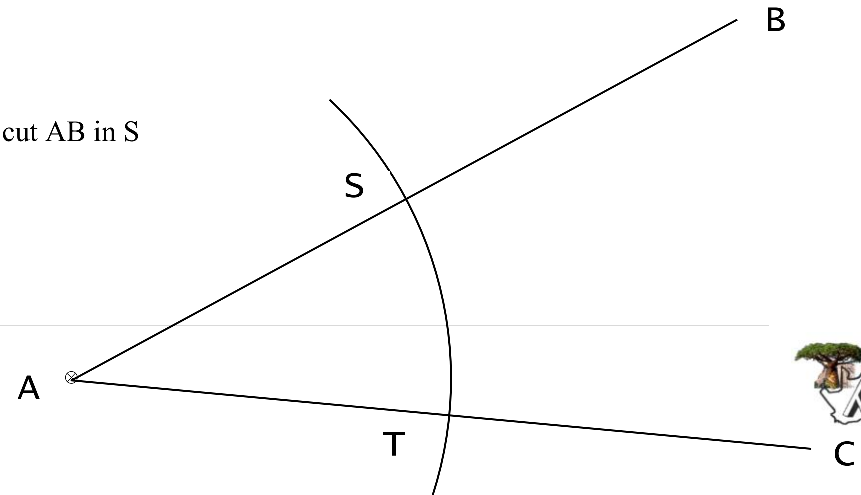
5. Investigate (a) whether PQ is the perpendicular bisector of AB, and

    (b) whether $\angle$ APQ = $\angle$ BQP.

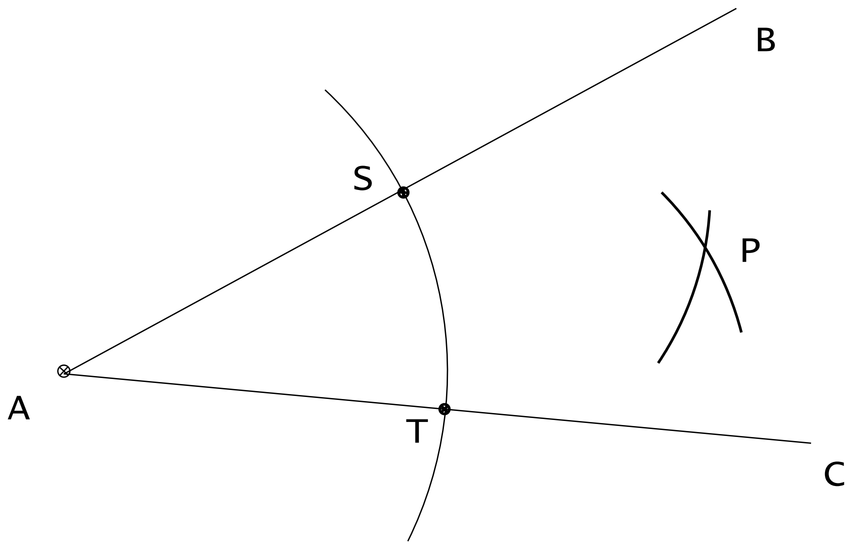
Activity 6: Use compasses to bisect a given angle

1. Follow these steps to bisect angle ABC on the next page.

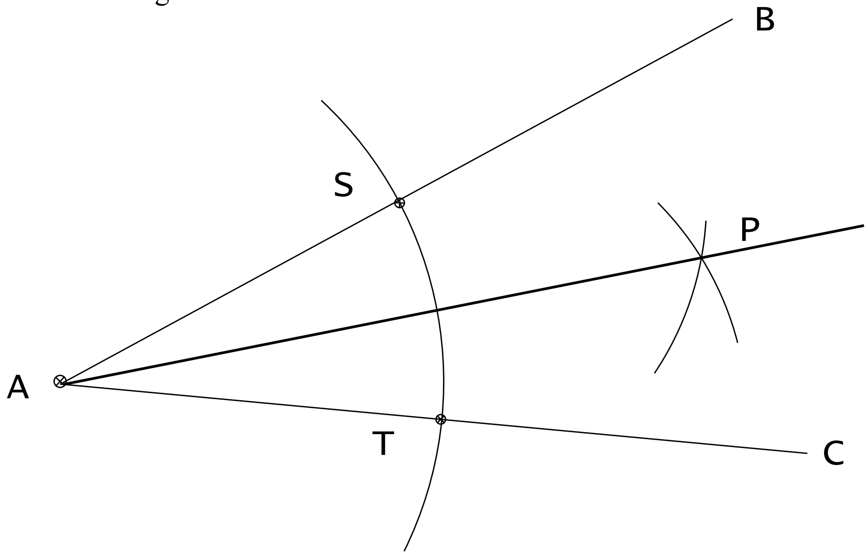
Step 1:
Draw an arc to cut AB in S and AC in T.

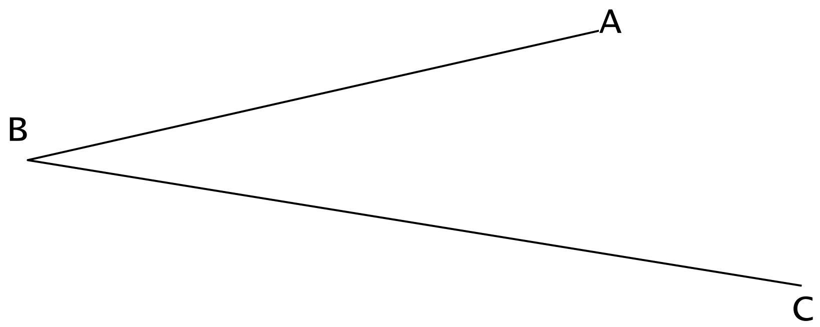


Step 2:
Draw equal arcs with S and T as centers, to intersect in P.



Step 3:
Draw a line from A through P.





2. Join A and C in the above figure, then use compasses to construct the angle bisectors of the

interior angles at A and C.

3. Draw any other triangle, then construct its angle bisectors.

4. Use your compasses and ruler only to construct a parallelogram accurately. Do this on a clean sheet of paper and make your parallelogram big so that it almost covers the whole sheet.

   Use your compasses to construct the angle bisectors of the four interior angles of the parallelogram.

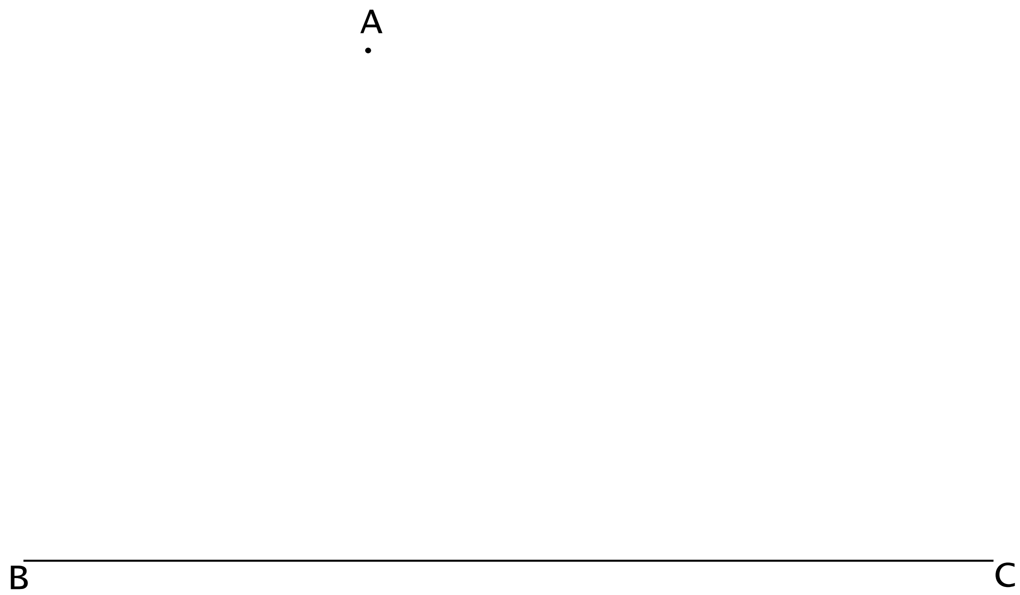
   If you observe anything of interest, report it to your teacher.

   At a later stage, you may produce a logical explanation of what you observed.

5. What kind(s) of quadrilateral has the property that its angle bisectors are concurrent? Investigate this thoroughly and write a neat report on a separate sheet of paper.

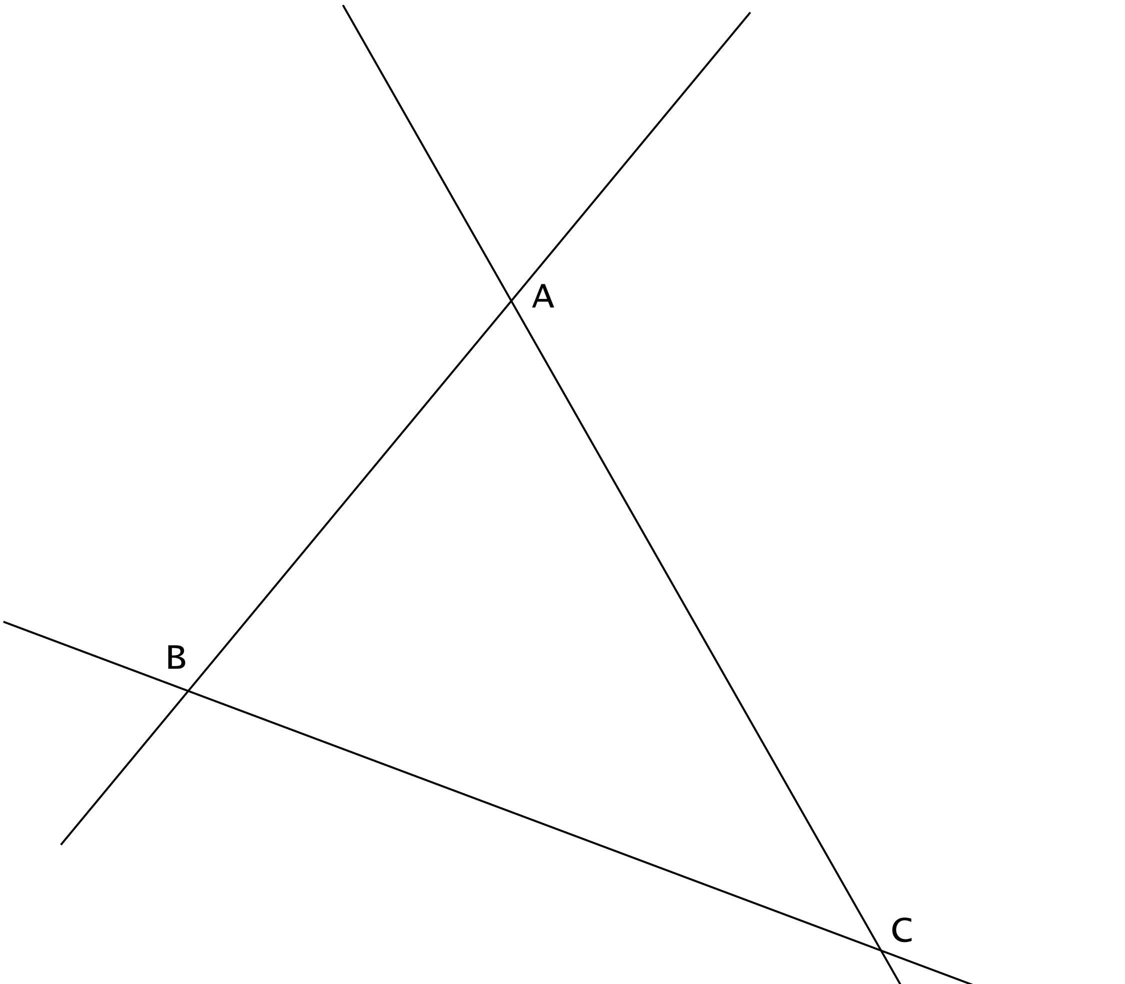
Activity 7: Construct a perpendicular from a point to a line

Step 1: Set your compasses so that an arc with center A will cut BC in two places at least 10 cm a-part.
Step 2: Draw such an arc, and mark the intersections with BC as P and Q.
Step 3: Draw a wide arc with radius about 7 cm and center P, below the line BC.
Step 4. Draw an arc with exactly the same radius as the one in step 3, with center Q, also below BC.
Step 5: Mark the point where the two arcs cut as T and join AT.



3. Construct the perpendicular from A to BC on the triangle below. Let the perpendicular meet

BC in P.



The line segment AP is called an *altitude* of the triangle ABC.

3. Construct the other two altitudes of the triangle ABC.

4. Are the three altitudes concurrent? Investigate this for another triangle.

PROBLEM SOLVING IN GRADES 8 AND 9

Susan Carletti and Jenny Campbell

The Answer Educational Publishers

Abstract

Are you a Grade 8 and 9 Maths educator? Are you wanting to help your learners develop their problem-solving skills? Are you looking for questions that you can use in your classroom that will extend your learners? If yes, then this workshop is for you. We will work through problem-solving questions, discussing various techniques that learners can be exposed to. In Grade 8 and 9, problem solving only counts 10% of any assessment. In FET, it counts 15%, so learners need to be exposed to techniques they can use from an early age. They also need to be encouraged to try problem solving questions, instead of shying away from them. The best way to gain confidence in problem solving is to do it in a non-threatening environment where you have time to play with the questions.

Target audience: Grade 8 and 9 educators

Workshop Duration: 2 hours

No. of participants: There is no maximum number.

Motivation for the workshop

I am passionate about problem solving and really enjoy helping others to see the fun in it. The joy you see on a learner's or an educator's face as they see the beauty in a particular problem makes teaching so satisfying. In this workshop we will work through problems, discussing skills that one can use to solve problems. These skills will help educators when they go back to their schools and can work through problem solving questions with their learners.

Description of content of workshop

We will work through 20 problems. The first ten will be problems that are based on syllabus work, so these can be used as Level 4 questions in assessments. The next ten problems are not directly related to any topic in the syllabus, but they will teach some problem-solving skills. These skills are essential to start learning as soon as possible, as they will help not only with Mathematics, but also any discipline that requires logical thought. The workshop will be hands-on, with educators working through the questions. The worksheet that will be used has been attached.

RE-IMAGINING IMAGINARY NUMBERS

Herman M. Tshesane

University of the Witwatersrand

Abstract

As in many other Mathematics topics, the traditional approach to teaching Complex Numbers tends to foreground algebraic processing over visualization. The problem with this approach is that it tends to perpetuate the ill-conceived notion that Mathematics is just a set of rules to be memorized and applied. In this 'How I Teach' session, I look to use a Transformations-based approach to provide Mathematics teachers with a visual interpretation of Complex Numbers, as well as the additive and multiplicative interactions between them.

Introduction

Since 2016, learners in Grade 10 have the option of taking Technical Mathematics, and this has enabled them to proceed through Grades 11 and 12 to Technical Colleges where they can register for N4. Also, in response to the skills required for the Fourth Industrial Revolution (4IR), the Department has also introduced new subjects. These include, Coding and Robotics, Earth and Space Sciences, Earth and Human Sciences, Aerospace Engineering, Biomedical Engineering, Ocean and Marine Engineering, and Entrepreneurship.

One topic that is indispensable to all these fields, and one that is now offered in the Technical Mathematics Curriculum, is the topic of Complex Numbers.

Problem statement

Traditionally, the approach to teaching Complex Numbers tends to foreground algebraic processing over visualization. The problem with this approach is that it tends to perpetuate the ill-conceived notion that Mathematics is just a set of rules to be memorized and applied. Moreover, if part of the national plan is to produce Aerospace, Biomedical, Software and Marine Engineers that can bring their ideas to life, part of the national imperative should be to provide these budding engineers with ways of visualising and communicating their ideas.

A Visual and Transformations-based Approach

One approach is to teach for learners' understanding of the relationships between complex numbers both algebraically and geometrically. This requires viewing complex numbers relationally: both numerically and graphically (Coles, 2017). Transformations Geometry not only provides us with a way of visually interpreting complex numbers, but also with a language that can support learners' conceptual understanding of complex numbers. it is stated in the Mathematics Teaching and Learning Framework for South Africa that:

> Conceptual understanding enables learners to see mathematics as a connected web of concepts. They should be able to explain the relationships between different concepts and make links between concepts and related procedures. Conceptual knowledge enables learners to apply ideas and justify their thinking (p.9)

To prioritise a conceptual understanding of Mathematics and to minimise "mindless symbolic manipulation", this "How I teach" session will provide teachers with a visual language for explaining complex numbers. Teachers will be initiated into the use of dynamic software that will, in turn, enable the use of the language of Transformations Geometry for explaining complex additive and multiplicative interactions between complex numbers.

Developmental outcomes

Teachers will come to understand that, for conceptual understanding, complex numbers should be viewed from four vantage points, thus:

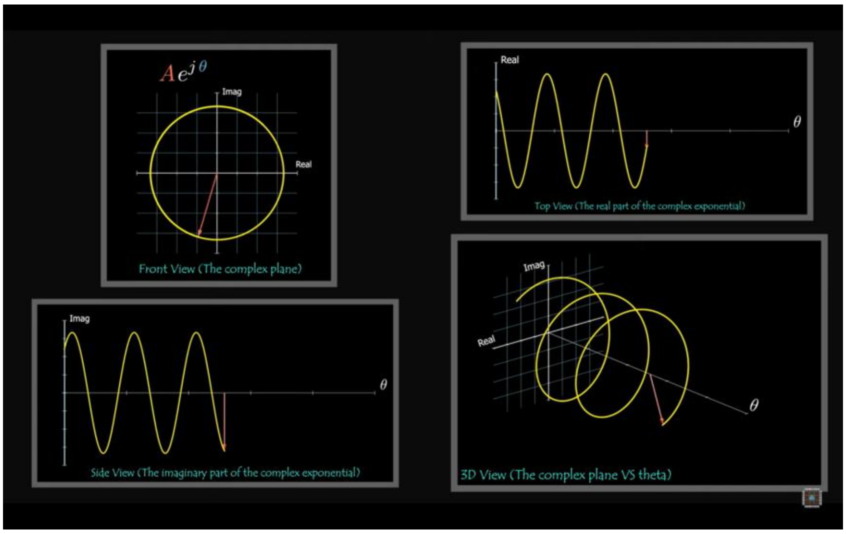
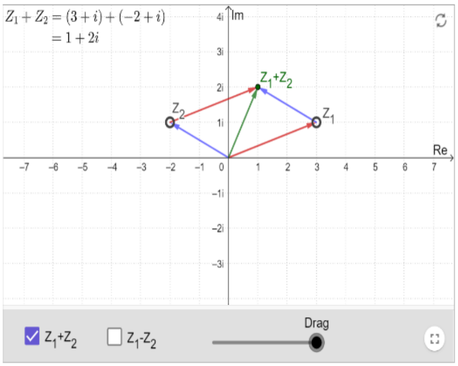
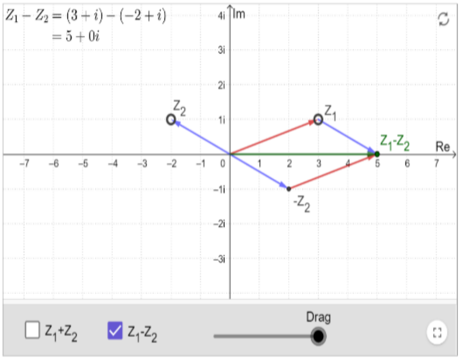
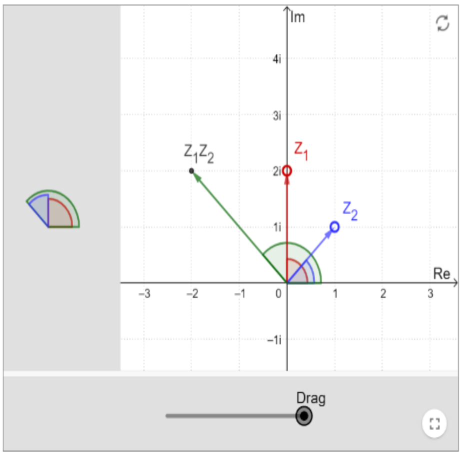


Figure 1: The complex exponential viewed from four different frames of reference

Operation	***Visual Interpretation***

<table>
<tr><td>Addition</td><td>Addition of two complex numbers can be understood as the addition of


two vectors using the parallelogram law. In addition, that the sum of two complex numbers is represented by the diagonal of the parallelogram formed by the two original vectors</td></tr>
<tr><td>Subtraction</td><td>

The easiest way to represent the difference $(Z_1 - Z_2)$ is to think in terms of adding a negative vector $Z_1 + (-Z_2)$. The negative vector is the same vector as its positive counterpart, only pointing in the opposite direction.</td></tr>
<tr><td>Multiplication</td><td>The geometric interpretation of multiplication of complex numbers in


the complex plane is stretching or squeezing for scalar multiplication, and; rotation for vector multiplication. Equally importantly, that to appreciate what happens geometrically we need to consider the polar form.</td></tr>
<tr><td></td><td></td></tr>
</table>

Conclusion

If learners are to learn Mathematics, and to understand it as a conceptually connected field of relational objects, then 'fostering a geometric transformations perspective means teachers must be [enabled] to engage learners in geometric thinking and encourage them to apply geometric strategies to solve problems, as opposed to solely arithmetic or measurement strategies' (Seago, Jacobs, Driscoll, Nikula, Matassa and Callahan, 2013, p.82).

References

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How I Teach Income Tax (Taxation)

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University of South Africa

INCOME TAX (TAXATION)

Teachers should take note that this is the only topic that is new in Grade 12 in Finance and thus needs to be taught thoroughly.
When unpacking the calculation of Income Tax for individuals, we should take the learners through a step-by-step method of calculating:

- Gross Annual Salary / Gross Monthly Salary
- Monthly Pension Fund /Annual Pension Fund
- Taxable Income
- Using a Tax Table to calculate the Annual Income Tax Payable before deductions.
- Tax deductions: Rebates and Medical Credits
- Annual Tax Payable after deductions

TOPIC: FINANCE					
Term	**ONE**	**Week**	**1 TO 3**	**Grade**	**12**
Duration	**15 Hours**	**Weighting**	**60%**	**Date**	
Section	**FINANCIAL DOCUMENTS & TAXATION**				
OBJECTIVES					
By the end of the lesson, learners will be able to do the following; ➢ **Define terminologies** ➢ **understand different types of financial documents, such as:** **Tax forms (IRP5, Employees income tax forms, tax deduction and tax rate tables)** ➢ **Understand the terminology used in these documents** ➢ **Identify and perform calculations involving the respective financial documents.** ➢ **Follow the steps in calculating tax payable by an employee or PAYE ,monthly and annually**					

RELATED CONCEPTS/ TERMS/VOCABULARY	
• **Gross income** • **Taxable income** • **Tax threshold** • **Tax rebates:** • **Medical tax credits:** • **Taxable deductions**	• **Non-taxable deductions** • **Net Pay** • **SARS** • **VAT** • **UIF** • **Pay slips**

PRIOR KNOWLEDGE/BACKGROUND KNOWLEDGE
To master this topic learner must have prior knowledge of: VAT Financial Document- Pay slip Terms like Gross salary, net salary ,basic salary, deductions etc. were used in Grade 10 &11 Calculating percentages Able to work with fractions and decimals Understand the concept of rounding Financial documents are part of our everyday life. In this section, you will work with and make sense of these documents under various contexts relating to your personal life, the workplace or a small business environment.

EXAMPLE

Rates of tax for individuals 2020 tax year (1 March 2019 - 29 February 2020)

Taxable income (R)	Rates of tax (R)
1 – 195 850	18% of taxable income
195 851 – 305 850	35 253 + 26% of taxable income above 195 850
305 851 – 423 300	63 853 + 31% of taxable income above 305 850
423 301 – 555 600	100 263 + 36% of taxable income above 423 300
555 601 – 708 310	147 891 + 39% of taxable income above 555 600
708 311 – 1 500 000	207 448 + 41% of taxable income above 708 310
1 500 001 and above	532 041 + 45% of taxable income above 1 500 000

Tax Thresholds

Age	
Under 65	R79 000
65 an older	R122 300
75 and older	R136 750

Tax Rebates

Tax Rebate	2020
Primary	R14 220
Secondary (65 and older)	R7 794
Tertiary (75 and older)	R2 601

Medical Tax Credit Rates for the 2020 tax year

Per month (R)	2020
For the taxpayer; or for a member or dependant of a medical scheme or fund where the taxpayer him- or herself is not a member of a medical scheme or fund	R310
For each additional dependant(s)	R209

Calculating Taxable Income

Example 1

Mrs Mkhize earns a monthly gross salary of R29 000. Calculate her taxable income.

Annual Gross Salary = $29\ 000 \times 12 = R348\ 000$

Monthly Pension Fund = $\frac{7,5}{\_\_} \times R29\ 000 = R2175$

or

100

(1750)

Annual Pension Fund $= R2175 \times 12 = R26\ 100$
Taxable Income = Annual Gross Salary – Annual Pension Fund

$= R348\ 000 - R26100$

$= R321\ 900$

Reading a Tax Table and Calculating Tax before Deduction

Example 2

Determine the tax payable before deductions for the following:

1. **A 60-year-old taxable income is R50 000**
2. **A 77-year-old taxable income is R120 000**
3. **A 62 old earning R170 000**

SOLUTIONS

1.According to the Tax Threshold, a 60-year-old whose income is R50 000 do not pay tax.

2. According to the Tax Threshold, a 77-year-old whose income is R120 000 do not pay tax

3. Tax before deduction $= 18\%\ of\ R170\ 000$

$= \frac{18}{100} \times R170\ 000$

$=$ **R 30 600**

Example 3 and 4

3. **Determine the tax payable before deductions for a 50-year-old earning a taxable income of R330 000.**

The taxable income falls in tax bracket 3 on the tax table.

Tax before deductions = R63 853 + 31% of taxable income above R305 850

$$= R63\ 853 + \frac{31}{100} \times (R330\ 000 - R305\ 850)$$

= R 71339,50

4. **Determine the tax payable before deductions for a 48-year-old earning a taxable income of R425 000.**

The taxable income falls in tax bracket 4 on the tax table.

Tax before deductions = R100 263 + 36% of taxable income above R423 300

$$= R100\ 263 + \frac{36}{100} \times (R425\ 000 - R423\ 300)$$

=100

875

CALCULATING DEDUCTIONS

There are TWO deductions that you will be making:

- **Rebates**
- **Medical Credits**

Rebates is dependent on the individuals AGE

Medical credits is deducted when the individual contributes towards a medical aid.

Tax Rebate	2018/2019
Primary	R14 067
Secondary (Persons 65 and older)	R 7 713
Tertiary (Persons 75 and older)	R 2 574

Per month (R)	2020
For the taxpayer; or for a member or dependant of a medical scheme or fund where the taxpayer him- or herself is not a member of a medical scheme or fund	R310
For each additional dependant(s)	R209

Example 1

Calculate the tax deductions for the following:

1. A person who is 48 years old and is married and have 1 child and pays towards a medical aid.

Rebate = R14067 (qualifies for primary rebate only) Why?

Medical Credits = (2 × R310 ×12) + (1 × R209 × 12)

= R7440 + R2508

= R9948

Total Deductions = R14067 + R9948

= R 24015

Example 2

Buhle is 55 years old and earns a gross salary of R674 000 per year. She has a husband and 3 children on her medical aid. Calculate her Annual Tax payable.

Step 1: Work out the Taxable Income

Monthly Gross Salary = $R674\,000 \div 12 = R56166{,}67$

Monthly Pension Fund = 7,5% × R56166,67

= R4212,50 (or R1750)

Annual Pension Fund = $R4212{,}50 \times 12$

= R50550

Taxable Income = $R674\,000 - R50550$

= R 623 450

Step 2: Work out the Tax payable before deductions using the Tax Table

Tax Before Deduction = 147 891 + 39% of taxable income above 555 600

$$= R147\,891 + \frac{39}{100} \times (R623450 - R555\,600)$$

= R 174 352, 50

3 children

Step 3: Work out Deductions

X 12 for 1 year

Deductions = Rebate + Medical Credits

= R14220 + (2 x R310 x 12) + (3 x R209 x 12)

=R 29 184

Main member +

Step 4: Calculate the actual Annual Tax Payable

Annual Tax Payable = Tax before deduction – Rebate – Medical Credits (Deductions)

= R 174 352, 50 – R29184

= R145168.50

Taxation involving DONATIONS (non-taxable)

Zweli who is 54-year-old is an employee at a Chemical Company. He earned an annual income of R367 000, including bonus which is equivalent to the monthly income for the 2018/2019 tax year.

- **He contributes 7,5% of his basic salary to the pension fund.**
- **He is also donating R 35 900 per annum to the registered charity organisation, the donation is tax deductible.**
- **He contributes R4 550 to the medical aid monthly, for himself and his 2 children.**

1. **Calculate the Taxable Income**
2. **Determine his annual medical credits**
3. **Hence, calculate his annual income tax**

In order to work out Taxable income, we must use the following formula:

1. **Taxable Income = Annual Gross Salary – All non-taxable items (Pension Fund and Donations**

Monthly Gross = $R367000 \div 13 = R28230.77$

Monthly Pension Fund = $\frac{7.5}{100} \times R28230.77 = R2117.31$

Annual Pension fund = 12 × 2117.31

= R25407.72

Taxable income = R367 000 – (pension fund + donation)

= R367 000 – (R25407.72 + R35 900)

= R305692.28

2. **Medical Credits = (R310 x 2 x 12) + (R209 × 12)**

= R 9948

3. **Annual Income Tax = 35 253 + $\frac{26}{100} \times (305692.28 - 195\,850)$ – Rebate – Medical credits**

- R14 067 – R9 948

= R 39 796.99

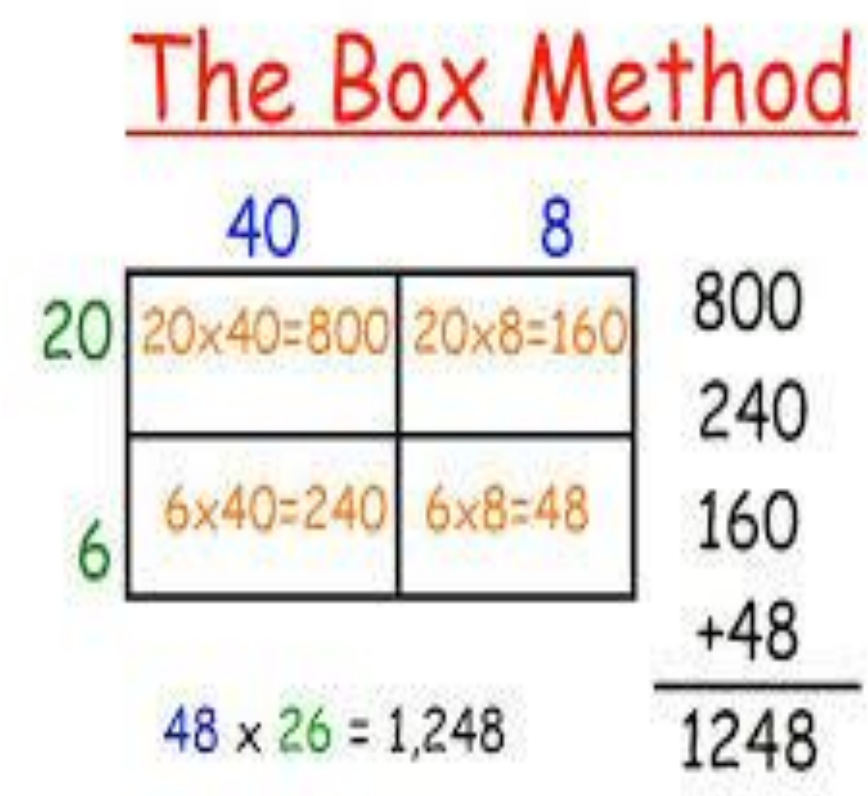
ACTIVITIES/ASSESSMENTS

THANK YOU

MULTIPLICATION SIMPLIFIED USING THE ARRAY METHOD AND NUMBER EXPANSION

Lebohang Sawukazi

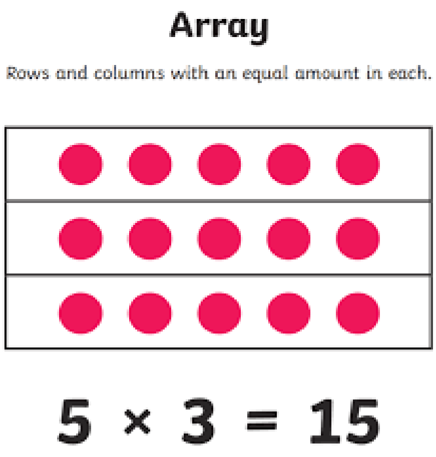
Bhekilanga Primary School



SIMPLIFYING MULTIPLICATION AND DOING AWAY WITH ROTE LEARNING

The array method of multiplication uses visuals to arrange objects or symbols into rows and columns. Now here's a quick example. Do you remember how we could easily add 3 apples to 2 apples and get 5? The idea is to do the exact same thing, but this time around, to teach multiplication.

In this case, the number of rows and columns represent the factors, and the total number of objects represent the product. Look closely at the example below:



In the Intermediate Phase, this method is taken a step further. Since the learners should have already been exposed to whole numbers, place value and expanded notation, why not use it to our advantage?

The following example is termed the Open Array, wherein larger numbers are broken down into smaller, more manageable parts (expanded notation). Below, follows an example of 230 x 12=……

Open Array:

- 230 is broken down into 200 and 30
- 12 is broken down into 10 and 2
- Create a rectangle with 2 columns (for 200 and 30) and 2 rows (for 10 and 2)
- Multiply each part:
  - 200 x 10 = 2 000
  - 200 x 2 = 400
  - 30 x 10 = 300
  - 30 x 2 = 60

X	200	30
10	2 000	300

2	400	60

Add all the products: 2000 + 400 + 300 + 60 = 2760

Why were arrays invented?

Arrays were originally invented to manage and organize data efficiently. Although this method is currently associated with programmers and other big terms such as Mathematical Vectors and Matrices, it has proven to be of good use is my Mathematics lessons as a student teacher back in the day, and for my learners currently.

what role does this method play in the classroom?

The array method has proven to eradicate rote learning methods that we often rely on to teach Mathematics concepts. These are often easy to remember as they are memorized techniques that are based on repetition and are not really about understanding the core concept that needs to be taught.

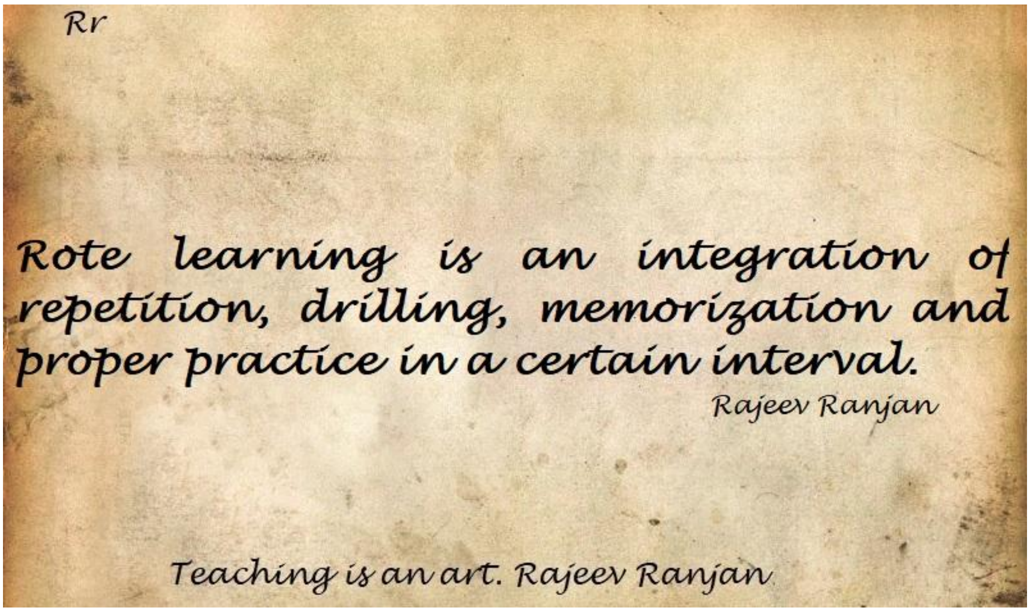
This methodology provides a clear relation between what the learners were taught about the nature of whole numbers and the relations between Mathematical operations. It has proven to be much easier to not treat each concept in isolation, but rather provide a clear image on how it relates to what was previously learnt.

The learners to have to ‘’borrow’’ or ‘’carry over’’ any numbers which they know no origin of. They simply rely expanding big numbers into smaller bit and their prior knowledge of multiplication also being known as repeated addition.

what the talk will entail.

In this talk, we will make links between the learners’ prior knowledge of whole numbers and what is being taught in multiplication.

The use of visuals will be encouraged as most teachers in public schools are faced with diverse learners, some even facing scholastic and learning barriers. This concept would make interesting intervention activities.


Rr
Rote learning is an integration of repetition, drilling, memorization and proper practice in a certain interval.
Rajeev Ranjan
Teaching is an art. Rajeev Ranjan

HOW DO I USE BASE 10 BRICKS, PLACE VALUE CARDS AND ABACUS TO TEACH NUMBERS AND DATA HANDLING: FOUNDATION PHASE (R-3)

Neo Nathan Ndaba and Marcus Maxwell

Bhekilanga Primary School

Introduction

Teaching numbers has always an easy subject for us teachers, but unknowingly we find ourselves having to deal with some learning losses because of learners do not understand numbers full. From decomposing numbers, number names, number symbols and lastly how to add, subtract, divide or even multiply. I have alluded to creating different methods to be able to successfully deliver number lessons to learners fully without leaving any learner behind. During the talk it is going to be hands on activity where all teacher engages with the presenter using those different manipulatives to demonstrate how best can they be used to teach numbers and data handling.

Content

Number decomposition and composition (Using abacus, place value cards and base ten bricks)
When I approach this topic its best to have my base ten bricks as they will be a visible manipulative to best assist learners to be able to understand Units, Tens and Hundreds. Although there are some disadvantages as some learners struggle with numbers or number recognition, nevertheless some find it as helpful as this manipulative help them understand numbers better and they know the nature of each number given to them as an activity

Number decomposition activity: Simply ask learners to set the given number on their abacus in any manner as they want Example: Learners may be asked to set the number 23 on the abacus and then monitor if they have set the correct number of beads of the abacus, after that ask the learners to place a decomposed number of 23 (as in 20 and 3) on their table using place value cards.

Number composition activity: From the previous activity the teacher might continue with the same number and ask learners to firstly clear the abacus then set the number 23 again and then after through monitoring of learners they must processed with to compose the number 23 with their table with their place value cards. Remember that when constructing /composing numbers the unit must overlap on the zero (0) of the 20 so that it makes one number.

Using base ten bricks Activity: Base ten blocks have different colours that represent different number values, so learners tend to understand better when you use them for number composition and number decomposition, Now ask learners to show you blocks that represent units, then tens, then hundreds lastly thousands. Then as learners to assemble a simple number just like 12 for starters, that will be a rod (tens-number block) and 2 blocks (units-number block) One of the disadvantages will be learners placing 12 blocks rather than one rod and two blocks. That depends on how the introduction of the lesson is done, So the teacher must insure the learners understand and know the representation of each base ten block.

• Using base 10 blocks has been effective according to the experience in the class as learners are able to classify different values of number value as they engage more and use the base ten bricks to assist them to solve problems given to them.

It is now then you can proceed to the topics like addition, subtraction, division and multiplication using all or some of the manipulatives to expend their knowledge, given that they will be having the basic knowledge of number and their nature.

Using an abacus to teach data handling

Utilizing this manipulative does not only work on numbers but what I have realized is that it can be used to collect, analyze and present data, one of the advantages is that learners will get the correct data that has been collected and the abacus can also be used as a representation of data (A bar graph) namely.

Our experience using an abacus to collect data has been one of the eye opening one as learners had more time to be hands on rather than doing activities during a lesson on paper.

Activity: Ask learners to place their abacus flat on the table while it is placed horizontal, beads bust be at the bottom of the side facing them. Now start asking them of the things that are placed in front of them. Example: Teacher may use different six bricks and ask learners to collect data of different colours of the bricks while they record each colour on their abacus. After that they will be able to see the number of bricks per colour represented of the abacus.

Conclusion

Numbers are easier if learners are allowed to be hands on, this activities are hands on activities. Providing learners more time to adjust and know numbers better. Coming to data handling I think it is important to install that assurance on learners and let them collect data on by themselves. In that manner you will be allowing them to be able to understand how to collect data, analyze data and represent data using a manipulative like an abacus. Hands on activities are basically a way to improve learner and teacher engagement so all the activities are learner teacher based whereby the teacher just guides the learners and let them make mistakes and learn from them.

Teacher tips

From experience most teachers do not use these manipulatives effectively, or rather they limit learners to learn using this manipulative. At times, the error that is made mostly is that teachers do not allow learners to discover thing on their own and that hinders learners to be able to be free to learn hence numbers is the most challenging topic. What I would advise is for teachers to model learning and allow learners time to discover thing on their own so that they are stimulated to learn even more and enjoy doing mathematics in the foundation phase.

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INTRODUCTION OF COMMON FRACTIONS USING OBJECTS

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MOTIVATION

Conceptual understanding of common fractions is very important when teaching fractions. Common fractions are seen as an easy topic by teachers and learners though it is not because learners are failing, and struggles as they proceed to other grades. The foundation is very crucial and it must be solid so as learners don't struggle in future, and how they tell fractions is really essential and basic as it builds on understanding fractions. Topics in mathematics must be taught in such a manner that learners don't forget, they must have knowledge of each topic so that they master it forever. Teachers and learners must strive for quality education not quantity, and the only way to make sure that we have quality education is through the best methods of instruction. Learners deal with fractions on a daily basis ,it can be sharing money, sharing food, telling time, for measuring and construction, discounts etc.. and it enhances their understanding because it deals with real life situations, when you relate maths with real world examples it makes sense and the topic becomes fun, learners engage and as they play they learn.

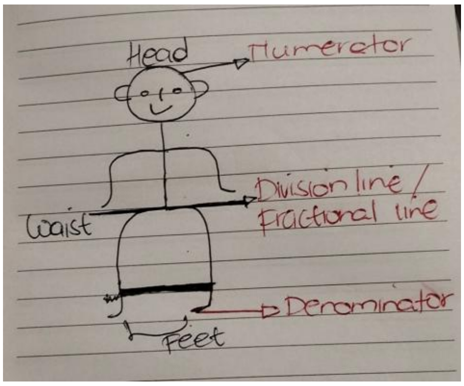
INTRODUCTION

Van de Walle er.al.(2016) emphasize the importance of the language used in learning fractions. To prevent misconceptions they advise that in the acquisition of the concept learners should be able to say, as in example for instance 'My whole is a rectangle. It is divided into six equal parts. Each part is a sixth of the whole conversely, given the whole (undivided) rectangle, they should be able to explain that " to find one sixth of a whole, I divide it into six parts of equal size and shade one part. The shaded part is one sixth of the whole.

CONTENT

Examples of common fractions
Proper -numerator is less that the denominator.
Improper -Numerator is bigger than the denominator.
Mixed number - combination of a whole number and proper fraction representing a value greater than 1.
Decimal - the fractions in which the denominator is equal to 10 or multiples of 10 (such as 100,1000,10 000 etc..)
Common fractions becomes more easier when taught using manipulatives, rhymes and real life objects.
Common Fraction Rhyme creation by Me Mapulana Letshego.
Basics.
RHYME
I am common fraction...
I have a numerator...
at the top...Of the division line...
I am a common fraction...
I have a division line...
In the middle...

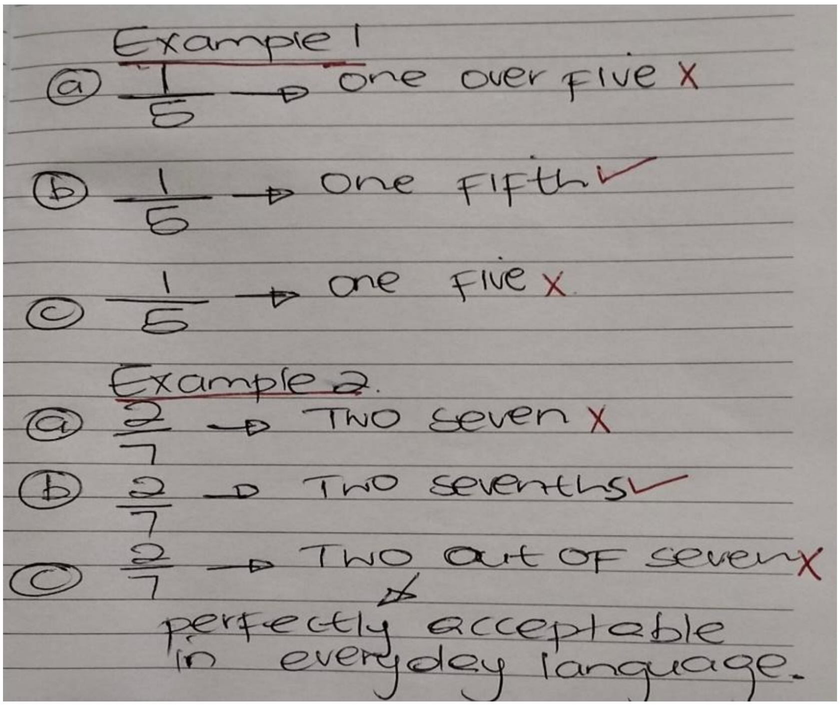
I am a common fraction
I have a denominator...
At the bottom...of a division line...
Denominator.....
D for down.



*As they sing they learn.
When teaching by use of real life objects there are steps that need to be followed.
Example : Share an apple equally amongst people.
Step 1: Number of items is 1 apple and it represents numerator.
Step 2: Count the number of people you are going to share an apple to (maybe it's 5 people)
Step 3 : Number of people represents the denominator
Step 4 : Use a knife to share five equal parts or pieces of apple for 5 people.
Step 5 : It means each person will get one fifth of apple.
Step 6 : when you add five one fifths the answer will be 5/5 and it is 1whole,which gives 1apple.
*Learners can join the equal parts of apple together again and they will see that they get full apple.

MISCONCEPTIONS

Learners and teachers sometimes tell fractions incorrectly and that makes it difficult for learners to understand fractions, if mathematical language is not mastered then it create misconceptions that lead to poor performance of results.
Example :
Telling fractions



In conclusion, a solid understanding of common fractions is exactly what learners need in order to compare, add, subtract, divide and multiply fractions. Math gives us hope that every problem has a solution, we must never give up.

Reference

Van de Walle, J.A., Kapok's & Bay-Williams,J.M (2016). *Elementary and middle school mathematics*. London : Pearson Education UK.

EXPLORING VISUAL MATHEMATICS FOR GRADE R

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Abstract

This presentation explores visual mathematics for Grade R, emphasizing relevance and engagement. The math adventure begins with an introduction to visual mathematics, highlighting its importance in developing problem-solving skills and promoting creativity. Through various activities, including pattern blocks, shape sorting, math scavenger hunts, number lines, counting games, math bingo, math story problems, and math art, students will engage with math concepts in a fun and interactive way.

Presentation Activities

The activities are designed to promote critical thinking, creativity, and collaboration, making math more accessible and enjoyable for all learners. Math story problems and math art activities encourage students to apply math to real-life situations and express their creativity. The presentation culminates in an assessment of problem-solving techniques, where students demonstrate their understanding of visual mathematics.

Ultimately, the math adventure concludes with a celebration of students' completion, acknowledging their growth and proficiency in visual mathematics and problem-solving techniques. The presentation aims to make math a positive and engaging experience for Grade R students, fostering a lifelong appreciation for mathematics.

CAN A STAR CONNECT FUNCTIONS?

Siya Nyathi

Zakhele Primary School, KwaZulu–Natal

Introduction

In a traditional mathematics classroom, there is a need to encourage learners to think more deeply on mathematical concepts, relate them to the real world situations around them, to avoid over emphasizing mathematical rules and procedure and attempt problem solving using as many representations as they can at the eighth grade level to make the learning more meaningful and lasting. The traditional instructional strategies should be modified in such a way that they include multiple representations as well as flexibility to use translation processes in representations (Oylum, 2004). One representation cannot describe a mathematical concept fully and since each representation has different advantages, so the use of a variety of representations may form the core of mathematical understanding (Ainsworth, 2004).

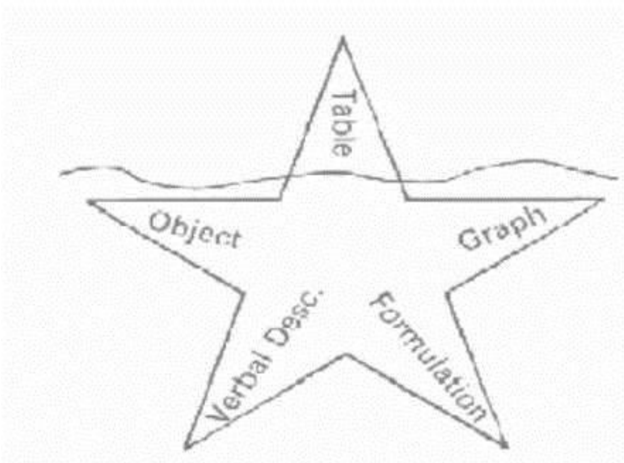
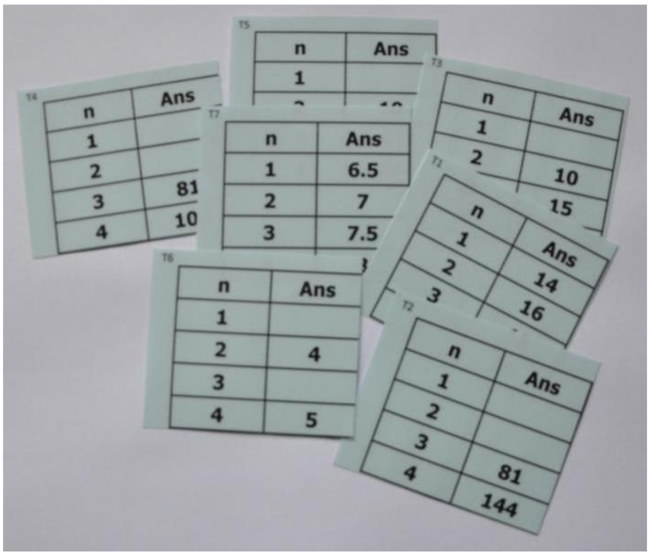
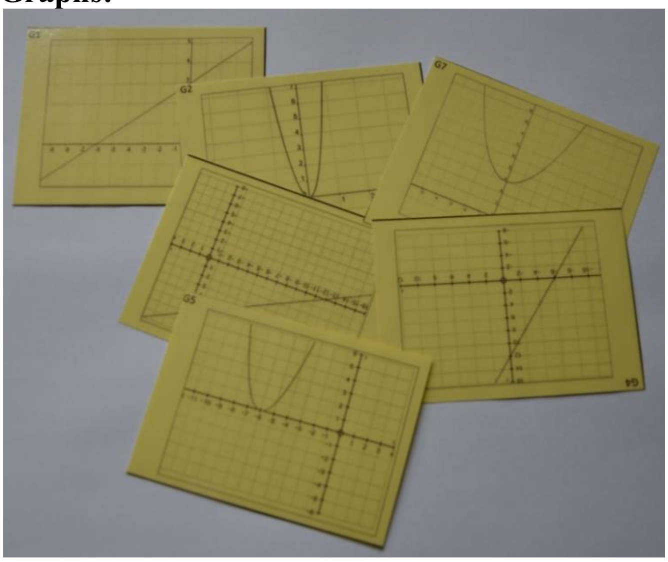


Figure 1 Janvier's Translation Model (1987)

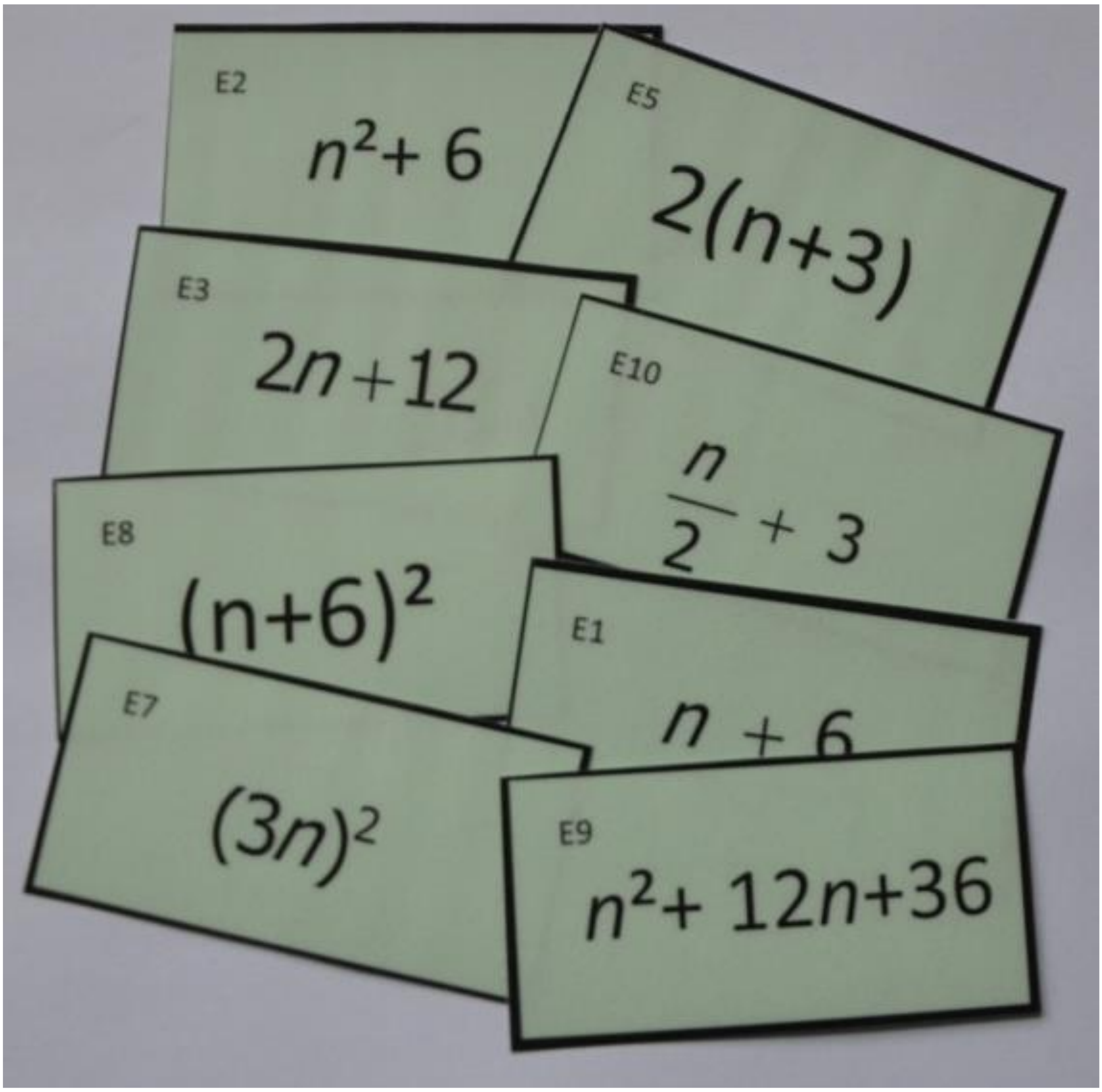
Tables:



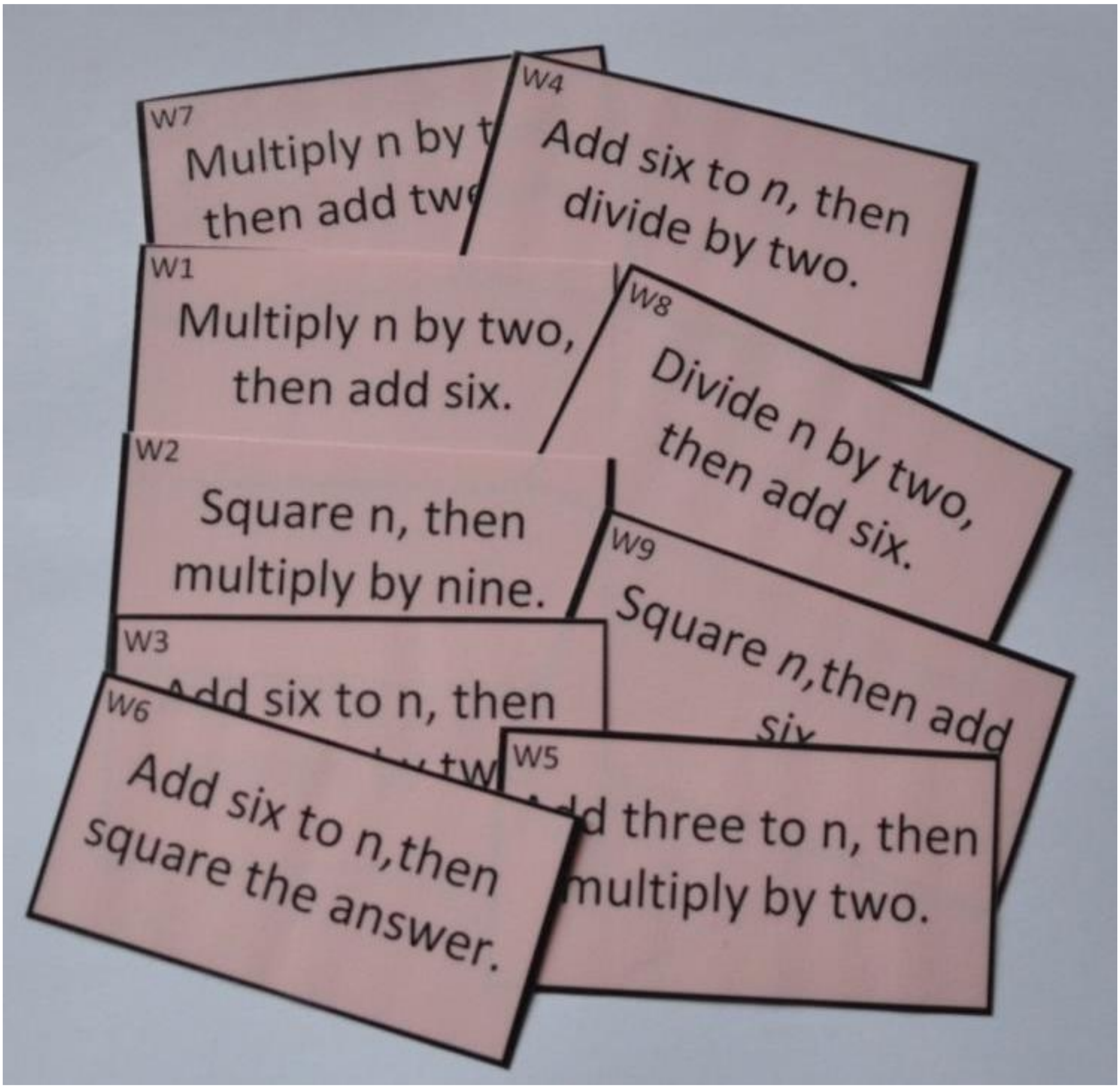
Graphs:



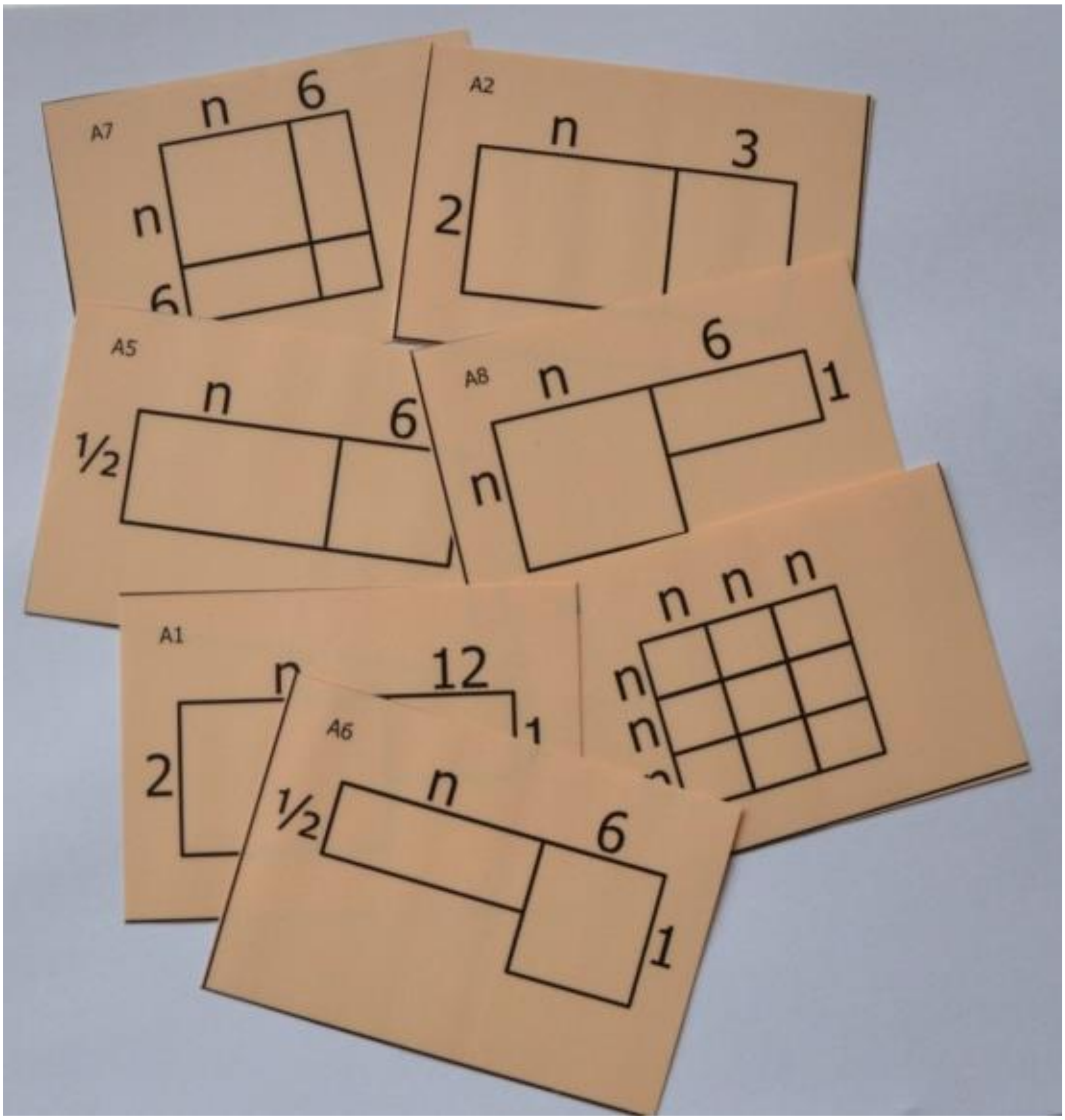
Formulation:



Verbal Discription:


W7
Multiply n by
then add
W4
Add six to n, then
divide by two.
W1
Multiply n by two,
then add six.
W8
Divide n by two,
then add six.
W2
Square n, then
multiply by nine.
W9
Square n,then add
W3
six to n, then
W6
Add six to n,then
square the answer.
W5
three to n, then
by two.

Object:



According to Janvier (1987) a representation may be considered as a combination of three components: written symbols, real objects and mental images. He greatly emphasized the external representations and made a visual resemblance between a representation and star (Figure 1) which shows the five kinds of external representations: Tables, Graphs, Formulation, Verbal Descriptions and Object. In Janvier's Model, the translation occurs while going from one vertex to another. Translation refers to psychological processes involved in going from one form of representation to another for example from an equation to a graph or vice versa and thus a translation always involves two forms of representations.

Conclusion:

This lesson has documented that compared to conventional mode of instruction multiple representations based instruction did make a significant contribution to the performance of eighth grade learners in functions. One representation cannot describe a mathematical concept fully and since each representation has different advantages, so the use of a variety of representations may form the core of mathematical understanding. The findings of this lesson indicate that using multiple representations, conceptual knowledge connections could be enhanced in a complex mathematical topic like functions. Yes, a Janviers (1987) star model can connect functions in a mathematics lesson.

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WHEN THE LINES OF SYMMETRY MEET VAN HIELE

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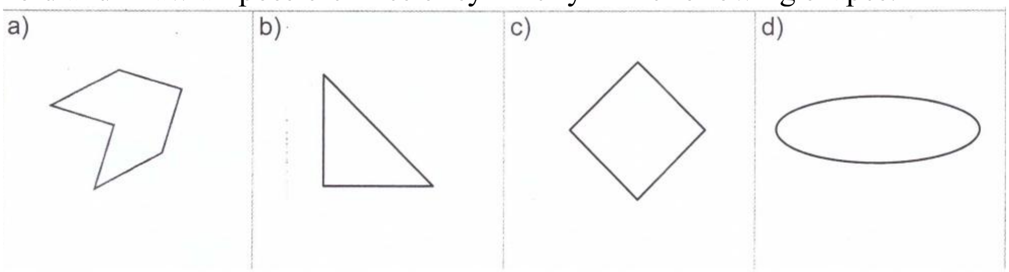
The purpose of this lesson is to re- examine the van Hiele theory of levels of geometric thinking and to compare this theory with the geometry curriculum recommended by the Department of Basic Education. The five phases of instruction are described and illustrated by examples dealing with the concept of symmetry. For Grade 7 classes, we will stop at the third phase.

First Phase: Information or Inquiry

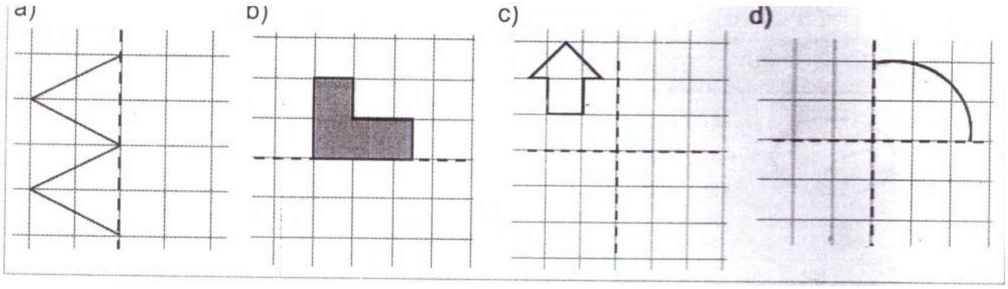
Fold and Draw all possible lines of symmetry in the following shapes:



Teaching and learning aids related to the current level of the lesson are presented to the learners. The teacher asks questions to the learners, introduces vocabulary words and allows the learners to make observations.

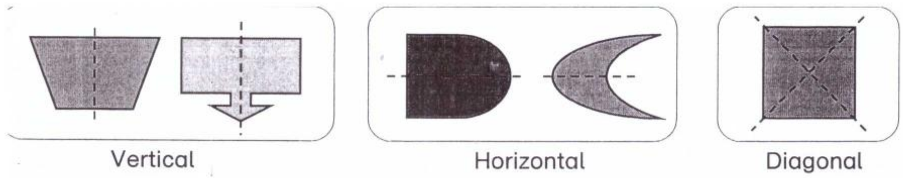
Second Phase: Bound Orientation or Guided Instruction

Using mirror, show how these reflections can be drawn:



The learners explore the field of inquiry through carefully guided, structured tasks. Learners are given examples to work with. Learners reflect the given line segments and determine what shape results. After completing all four reflections, learners make observations about the line of symmetry of the

Third Phase: Explication



The learners with the teacher engage in discussion about the objects of study. Language appropriate to this level is stressed, and clear up any misunderstandings so far. A teacher can switch codes in order to translate or clarify instructions, but also to reformulate and model appropriate mathematical language use. Now the learners can say that lines of symmetry may be vertical, horizontal and also diagonal depending on the reflection of the shape.

According to the van Hiele model, each learning period builds on and extends the thinking of the preceding level. Effective learning occurs as the learners actively experience the objects of study. Language is an integral part of learning. New language is introduced in each learning period to make explicit and discuss new objects of study. My Grade Sevens did wonders in Term 2 assessment in the lines (axes) of symmetry.

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Using Lesh Translation Model to Connect Fractions.

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Introduction

Fractions are well known to be difficult for learners to master, to the extent that De Turck, (2008) argued that they should be scrapped from the curriculum. He maintains that learners in the primary school are no ready to grasp the concept of fractions because it is too abstract. In his opinion the problems learners experience are due to the methods used by teachers such as emphasizing algorithmic procedures (e.g. invert and multiply) without any understanding by learners of why this is appropriate. For my lesson I will use the Lesh's Translation Model (1983). Lesh's model emphasizes the use of five modes of representations: *spoken symbols*, *real world situation, manipulative, pictorial representations, and written symbols*. Lesh (1979) defined the ability to translate from one mode of representation to another as related to mental processes and abilities and differentiated between within-mode translation processes and across-modes translation processes.

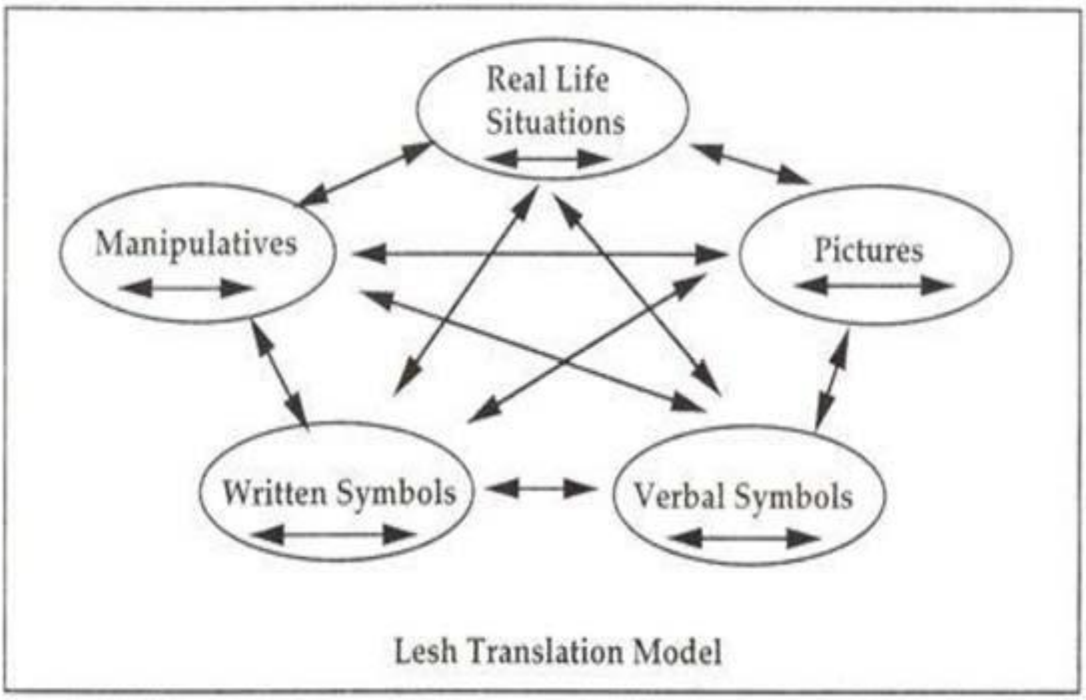

Figure 1 Lesh Translation Model (Lesh et al., 1987)

Real Life Situation:

Ntombi, Thandi, Pinky, Gugu and Thobile have 8 bars of chocolate that they want to share equally among the 5 of them so that nothing is left. How many pieces of chocolate will each child get?

To show that knowledge is organized around real life events.

Manipulative
These are objects that can be touched, moved and often stacked like fraction circles, unifix cubes, Cuisenaire Rods, etc.

Pictures

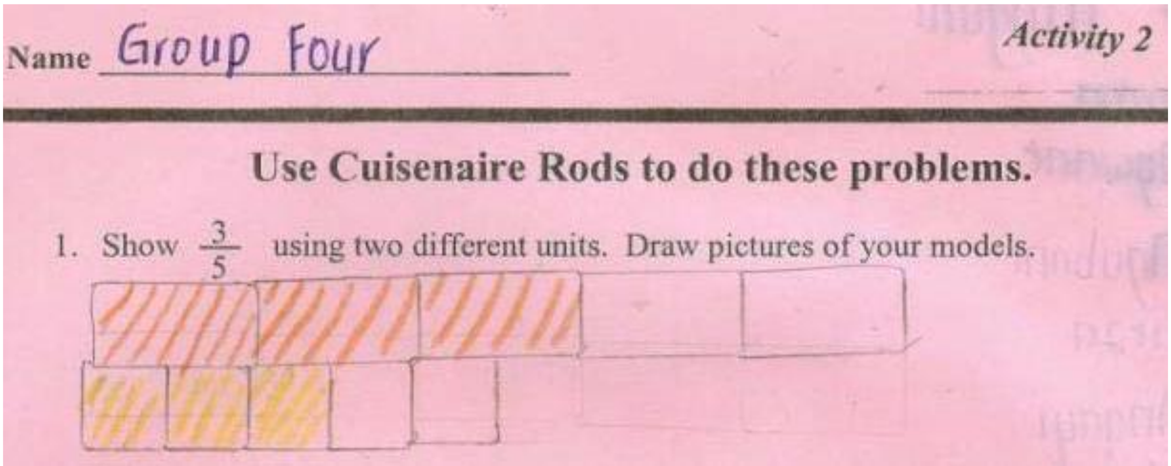

Figure 2 Picture of Activity 2:
Pictures are figural modes or diagrams.

Spoken Word
Cuisenaire rods are bars that use length and color to represent fractions. They were used to demonstrate equivalent fractions, shown by trains (rods line up end to end) of the same length. There may be several groups of equivalent fractions for each unit. The numerator of each fraction is the number of rods used in the fraction. The denominator of each fraction is the number of rods that are used if the train is equal to the unit.
This can be everyday language that can be used in the teaching and learning situation.

Written Symbols

a. *Into how many 1/6 can you cut 14 oranges?*

These are mathematical symbols, words, phrases that can be utilized in the mathematics classroom.

Conclusion
My three Grades 7 classes performed exceptional well in their third and forth terms assessments that had fractions as part of the assessment after teaching them using the Lesh Translation Model. The importance of the thorough understanding of fractions for the development of further mathematics proficiency is illustrated by looking at the other aspects of mathematics. The entire study of linear equations is dependent on the slope of a line, a fraction representing the rate of change. To solve rational expressions it is necessary to apply generalized fraction concepts. Solving quadratic equations by completing the square also requires fluency with fractions.

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HOW I TEACH MULTIPLES-A REMEDIAL GRADE 7 LESSON

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Introduction

The presentation is on how to assist grade 7 learners who are struggling with their multiplication tables s\o they can better remember using patterns. I have realized that some grade 7s have not mastered their tables and senior phase requires a lot of multiplication without using a calculator. Some learners end up giving up on the subject. They only do it because they must but there is no joy in it. In the presentation participants will try out the methods so that they can see the practicality of these patterns

Content

Involving the learners to formulate the patterns leaves a lasting impression. Instead of drilling them or taking them through a method bring them on board and ask them to identify patterns. A disadvantage to this approach would be a learner mixing up the method and thus ending up with the wrong multiples or worse forgetting how to generate the subsequent numbers. An advantage is once they identify the pattern they will own it and the answers to multiplication and related concepts will be correct. The confidence then breeds a love for the subject.

References

https://youtu.be/2F-t16Bm55s?si=M_2lXEwplFmTAaJY
https://youtu.be/9qQUCkCFozU?si=mvbXdotGbP9omr95
https://youtu.be/5_RZ6RjUmLo?si=ABMhy8c9Zy8k7SHW
https://youtu.be/mx_xqzyoJoI?si=suwZyVi5hVtXxmZo
Spot on Mathematics Poster Tricks with the 9x table, Pearson

TEACHING MULTIPLICATION AND DIVISION AND ENSURING NO CHILD IS LEFT BEHIND

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Introduction

South African classrooms are increasingly diverse, and many Foundation Phase learners struggle with basic numeracy. Grade 3, being the final Foundation Phase year, often reveals significant gaps in multiplication and division understanding. These concepts are foundational for success in Intermediate Phase mathematics, yet teaching them effectively remains a challenge. This proposal is motivated by my dual role as an educator and subject leader committed to inclusive education and equitable mathematics outcomes.

Content

This proposal is to address the challenges and responsive strategies involved in teaching multiplication and division to Grade 3 learners in a diverse Foundation Phase setting. The presentation focuses on ensuring that no learner is left behind, particularly those with learning barriers and language challenges. Using different and effective methods for different learners with different learning styles **(visual learners, auditory learners, kinaesthetic and reading and writing learners),** lesson observations, learner work samples, and feedback from critical friends, the proposal explores inclusive and differentiated teaching practices that support conceptual understanding. As a female subject head and Mathematics coordinator, I reflect on my leadership role in mentoring Foundation Phase teachers and leading professional development for more equitable classroom practices. The presentation will contribute to curriculum dialogue, inclusive pedagogy, and leadership in early Mathematics education.

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BREAK IT, SHARE IT, OWN IT: DIVISION MADE EASY

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Abstract

This proposal investigates the use of interactive and hands-on methods for teaching long division in Grade 6. Many learners struggle with division due to its multi-step nature and abstract reasoning. This research supports the integration of visual tools, real-life contexts, and learner-centred techniques—such as songs, affirmations, and manipulatives—to improve understanding, motivation, and performance in long division. The proposal draws on recent research highlighting how experiential and gamified strategies promote deeper mathematical engagement, especially in the intermediate phase.

Content

Recent research by Smith et al. (2020) and Chen et al. (2021) indicates that learners benefit more from active, visual, and sensory-based instruction than from traditional lecturing. Long division, in particular, becomes more manageable when broken down using memory tools such as mnemonics (e.g., “Does McDonald's Sell Cheese Burgers” for Divide, Multiply, Subtract, Check, Bring down) and musical repetition.

Using real-life examples (like sharing money or splitting groups of items) connects math to learners’ lived experiences, while tools such as base-10 blocks, grid paper, and digital apps provide visual reinforcement.

Jackson & Lee (2023) emphasize that without proper alignment to curriculum goals and formative support, such methods may become distractions rather than tools. Thus, scaffolding and curriculum coherence are key to effective implementation.

Teacher Tips

- To successfully integrate learners' learning, start with basic manipulatives and gradually introduce more sophisticated tools.
- Encourage learners to take charge of their education by guiding research and making discoveries in interactive projects.
- Encourage thoughtful conversation and discourse to improve understanding and successfully dispel myths.

Some practical implementation tips and strategies for effectively integrating hands-on activities into the classroom:

1. **Lesson Planning Tips**
    - Align with CAPS goals: Ensure activities meet the content and skills listed for Grade 6 mathematics under the Numbers, Operations, and Relationships strand.
    - Use progressive scaffolding: Start with two-digit ÷ one-digit examples before progressing to complex long division with remainders.
    - Incorporate song and rhythm: Teach learners a division chant to internalize the algorithm (e.g. "Break it, share it, check it out!").
    - Create a visual anchor chart: Keep D-M-S-C-B steps visible at all times.
    - Use hands-on tools first: Use grouping blocks or counters before moving to symbolic notation.
2. **Assessment Strategies**
    - Observation: Monitor learners' steps during problem solving for conceptual accuracy.
    - Mini whiteboard checks: Have learners solve in steps and hold boards up for immediate feedback.
    - Peer teaching: Ask learners to explain steps to a partner.
    - Performance task: Create real-life problems (e.g. sharing R72 among 6 learners) where learners apply long division and explain their reasoning.
    - Reflection writing: Learners journal what they found easy or difficult in solving long division problems.

3. **Differentiation Techniques**
    - Multiple representations: Use song, visuals, manipulatives, and written steps to cater to varied learning styles.
    - Flexible grouping: Pair learners strategically for peer support.
    - Support stations: Create a "step support" table for learners who need more guidance with each part of the division process.
    - Challenge extension: Offer complex word problems or division for early finishers.
    - Visual scaffolds: Use coloured grids or boxes to help learners separate steps visually.

Conclusion

Teaching long division in Grade 6 presents a valuable opportunity to transform abstract mathematical procedures into engaging, meaningful learning experiences. By using interactive strategies—such as hands-on manipulatives, songs, real-life contexts, and scaffolded support—educators can make long division accessible and enjoyable for all learners.

This proposal advocates for a learner-centred approach that encourages exploration, repetition, and reflection, supported by recent educational research. With intentional planning, continuous assessment, and inclusive teaching techniques, teachers can empower learners to break it, share it, and truly own it—mastering long division with confidence and understanding.

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<u>The End</u>